Babylon: Linear Measures of Celestial Angles and an Observatory

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Abstract

The paper examines Babylonian records, from the 1st millennium B.C., of planets passing fixed stars and specifically their up/down differences in linear cubits. It shows they were using the top of a gnomon as a foresight around which the observer moved on non-circular arcs, where the ratio of degrees per cubit was 2.5° (azimuth). Particularly near the horizon they were able to ensure close alignment in longitude between the star and the planet. The up/down measurements were then almost identical to the distance between the two bodies, using a straight rule. Finally an area north of the Western Court of the Southern Palace is identified as a possible site of the observatory.

This study is based on the surviving Babylonian records of planets passing fixed stars in the years from –418 to –73 BC .[1] The collection includes 1049 passages, where the up/down differences were recorded in linear units with a maximum of 6 cubits and a minimum of 1 finger (1/24th cubit). There were also 27 records of similar before and after differences. The recorded information was laconic with an up/down report for the year –418 reading simply 'Month II, night of the 9th, Mars was 4 cubits below θ Leonis'.[2]

Not all the possible combinations of fingers and cubits are represented. There were certain preferred values with rounding errors running from +/- ½ finger, for distances under 6 fingers, to +/- 6 fingers for those above 4 cubits. In percentage terms such errors reach +/- 50% with a minimum of about +/- 3%. For distances below 1 cubit, the percentage is between 7 % and 50% and above 1 cubit between 3% and 8%.

There are gaps in the longitudinal coverage of the Normal Stars and, consequently, in declination. In terms of azimuth, the gaps, near the horizon, can be seen in Figure 2.

Professor Jones concluded that the up/down cubit values were related to differences in latitude and found the mean ratio of degrees (latitude) per cubit to be about 2.3°, which lies between the two ancient norms of 2° and 2.5°.

Ptolemy, in his criticism of the data, provided clues about how the measurements were made:

In general, observations [of planets] with respect to one of the fixed stars, when taken over a comparatively great distance, involve difficult computations and an element of guesswork in the quantity measured, unless one carries them out in a manner which is thoroughly competent and knowledgeable. This is not only because the lines joining the observed stars do not always form right angles with the ecliptic, but may form an

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[1] Jones, A., A Study of Babylonian Observations of Planets Near Normal Stars, Arch. Hist. Exact. Sci. 58 (2004) pp.475-536. I am very grateful for Professor Jones for giving me access to his Collection A and also to my son, Geoffrey, for help with the drawings and his patience.

[2] Sachs A.J. and Hunger H., Astronomical Diaries and Related Texts from Babylonia, Vienna 1988, Vol. I, p.63

angle of any size (hence one may expect considerable error in determining the positions in latitude and longitude, due to the varying inclination [to the horizon frame of reference]); but also because the same interval [between star and planet] appears to the observer as greater near the horizon, and less near mid-heaven;[footnote] hence, obviously, the interval in question can be measured as at one time greater, at another less than it is in reality.

[footnote] This appears to be the only reference to the effect of refraction (if that is what it is) in the Almagest, despite its obvious relevance to the observations of Mercury's greatest elongation….[3]

Clearly he considered they were thinking in terms of the ecliptic, but were also assuming lines of longitude and latitude were always at right angles.

To investigate how and what they were measuring two main assumptions were made:

1. The observer used a linear measuring rod to determine the up/down or before/after position of the planet in relation to one of the 28 so-called Normal stars. He did this by aligning one end with the star and the other point with the planet, using the top of a vertical gnomon as a foresight (Figure 1).[4] The rod may have been hand-held or fixed in a rest. For each passage, the ratio of degrees (latitude) per cubit was converted to DTOG, the distance of the observer's eye from the top of the gnomon.[5] The mean ratio of 2.3° per cubit implied that his eye was about 25 cubits from the top of the gnomon, which equates to ca. 13 metres and gives an idea of the size of the device, assuming a cubit of about 52 cms. Any errors in measurement and recording, including the not inconsiderable rounding errors, are accumulated within the DTOG value.

2. In 96% of the surviving records the difference in longitude (planet less star) was between –3.3°/+3.8°. Consequently the distance between the two bodies would be only marginally greater than their difference in latitude. By repeated iterations 656 passages (63% of the surviving records) were found where the distance between the two bodies was within 0.2% of the recorded up/down distance in cubits.[6] The margin of 0.2% is small but it equates to a 3.6° difference in the before/after positions, if a rectangular co-ordinate system was used, as Ptolemy indicated.[7]

Professor Jones calculated the ecliptic co-ordinates of the outer planets for midnight and those of the inner planets about 4 hours, either before or after midnight.[8] No adjustments were made either to these celestial co-ordinates or for refraction.

Appendix A has a worked example of the calculations for one passage.

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[3] Toomer G.J., Ptolemy's Almagest, Duckworth, London, 1984, p.121.
[4] The main justification for such an arrangement is that it brings the measuring scale close to the observer. It is not essential as the observer could be at the centre of the device with the scales about 13m away, but in that case it is hard to accept the 'measurements' as much more than estimates.
[5] DTOG, distance from top of gnomon, equals 1/sine(ratio degrees per cubit)
[6] Varying the altitude of the two bodies changes the linear distance between them. Out of the 128 passages with up/down distances of 4 fingers or less and rounding errors greater than 12.5%, only 29 had the distance apart within 0.2% of the recorded value.
[7] Using plane trigonometry, Cosine (3.6°) = 0.998.
[8] Jones A., op cit. p.481 gives UT 21 for the outer planets and either 17 or 1 UT for the inner planets.

Before and After Alignment

There has been considerable discussion about their ability to measure in ecliptic co-ordinates.[9] Of the 656 passages 59% were most closely aligned in longitude particularly at lower altitudes. In 78% of these passages, the longitude difference (planet less star) was less than 1°, compared with 63% of all surviving records. Other passages were better aligned in R.A. (34%) or even azimuth (7%) (Figure 2). This confirms Professor Jones's conclusion that they were thinking in ecliptic co-ordinates, but, it now appears, their alignments, in longitude, were closer at lower altitudes. The mean altitude of the stars for the passages, best aligned in longitude, was 7.5° and, for the others, 16.6°.

Positions of the Observer's Eye

The passages, best aligned in longitude, were sorted to the order of the star's azimuth, in the west and the east. The relationships between azimuth and, separately, the north/south and east/west cubit co-ordinates of the eye of the observer are shown in figure 3.

Surprisingly the relationship between azimuth and the north/south co-ordinates is very close to linear, with each cubit corresponding to 2.5° of azimuth, which implies that the paths of the observer were neither circular arcs around the gnomon nor straight lines.[10] Instead those paths must have been stepped arcs.[11] This provides a good indication of the intended paths of the observer in the east and west.[12] However, in practice, within 20° of due east/west, the divergence from a straight line is less than 1 cubit and could well have been ignored, if a straight line was more acceptable.

The observer's position in the vertical is generally within 6 cubits of the top of the gnomon, but drops to 10 or 11 cubits in places (Figure 4). There is a notable anomaly about 5 cubits north of the gnomon, where, particularly in the east, the observer's eye drops down to about 10 cubits. This anomaly also marks a sharp fall in the number of passages, when the observer is between 5 and 10 cubits north of the gnomon.

From the foregoing we can deduce that there was a structure around the gnomon which facilitated observations where the observer's eye was within 6 cubits of the top of the gnomon.

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[9] Hunger H. & Pingree. D. Astral Sciences in Mesopotamia, Brill, 1999, p.269.

[10] This also confirms that they were thinking in terms of 2.5° per cubit.

[11] I am most grateful to P. Starkey, a neighbour and mathematician, for providing the modern polar equation for such curves: $r \sin \theta = (Y_{max} .2/\pi)\, \theta$, where r is radius and θ the angle in radians. Spiral curves may have been used in Egypt at an early date (see Appendix B). A similar linear relationship, but closer to 2.6° (azimuth) per cubit, was found for those passages best-aligned in R.A., indicating that for those passages the observer was slightly closer to the gnomon. As they were also higher, it implies that the observer's path, in cross-section, was like a steep-sided bowl.

[12] A target ratio of 2.5° per cubit implies the distance to the top of the gnomon on the east/west line was about 23 cubits (1/22.9 = Tan 2.5°). From there it is simple to calculate thirty-six cubit steps, each of 2.5°, to the north and south. With due north/south being at 0,36/0,-36 and due east/west at ca. 23,0/-23,0. The intermediate positions at 45° are +/-18, +/-18. With 36 steps the sum of each successive hypotenuse totals 44.6 cubits, so along that path each cubit averages about 2°. In practice the steps may have been irregular and larger than the 1 cubit assumed. If used to measure altitude, rather than azimuth, such a curve would resemble the recumbent crescent moon, a common motif in Mesopotamia, but an impossible position for the moon in practice.

There are other aspects brought out by the moving mean lines in figure 4.[13]

A Possible 'Observatory'

To visualise the observatory, we might think of a 6 cubit gnomon standing above the flat roof of a building with the much lower areas corresponding either to the ground outside or to interior open courtyards. In the Southern Palace at Babylon, there are many such courtyards, but there is an area north of the Western Court of particular interest. Before the whole of the area had been excavated, a part to the north-east was described as follows:

The houses of this part of the palace are remarkable for the strength of their walls and the admirable regularity with which they are laid out. Court 38 is reached by a passage-way from the Principal Court, the latter through a hall, as in the case of 25, 26 and 27, opens with three doors on to court 38. Between the doors, pillars project from the walls and correspond with others on the opposite side. They must have served as piers to support arches for the ceiling, although it is difficult to make out clearly what was the object of this structure.

The roof of this area of the palace was evidently intended to support more weight than usual. It may appear improbable that an observatory would be rectangular, but we can perhaps think of it as being like graph paper. Today we use Mercator charts, with rectilinear lines of longitude and latitude, and also Ordnance Survey maps with a rectangular grid. It is a question of balancing the pros and cons of such arrangements.

There are circles in the sky which produce straight lines, aligned with the cardinal directions, on the ground. Firstly there is the meridian. Secondly a prime purpose of an observatory would have been the measurement of time both at night and during the day. In a horizontal sundial the hour-line for 6 hours to transit runs due west/east through the pole. Thirdly the shadow of the sun, at the equinoxes, runs due west/east just north of a gnomon.[14] We thus have three perfectly straight lines – the meridian, the hour-line for six hours to transit and the shadow of the sun at the equinoxes – and we have already noted that the stepped arcs run sensibly due north/south within 20° (8 cubits) of due east/west. Together these lines form a near rectangular outline for observations.

Just south-east of court 48 is a short length of wall of abnormal width (1.8m), which is aligned with a passage leading from the northern wall of the palace. None of the other similar passages, running due south, from the oblique northern wall, is so short.[15] The wide wall and the short passage may perhaps have marked the meridian.

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[13] Moving means help to smooth out erratic data, but depend on how the data was sorted. In figure 4 it was in order of N/S cubits, but in figure 6 in order of azimuth.

[14] With a gnomon of 1 cubit, on a latitude of 32.5°, the pole would be 1.57 cubits to the south and the equator 0.637 to the north, with the distance between them being 2.207 cubits.

[15] The oblique northern wall of the palace is stepped, both vertically and horizontally, and is inclined about 17° from east/west. The 17° of azimuth matches that quoted for the limits of the path of Anu in Walker C. (editor), Astronomy before the Telescope, British Museum Press, 1996, p.48. It corresponds to the rising/setting of stars with a declination of +/- 14.3°, which is close to the 15°, for the Path of Anu, quoted in Hunger H. and Pingree D., Astral Sciences in Mesopotamia, Brill, Leiden, 1999, p.61.

The short thick wall links two substantial east/west walls, about 5m apart; one just south of house 48 and the other north of the transverse corridor.[16]

The stepped curves, with each north/south cubit corresponding to 2.5° of azimuth, would fit within the north/south width of this part of the palace, with the gnomon about midway between the two east/west walls. However, as we will see, there are reasons to believe it was perhaps ca. 2m further north. In figures 5 & 6 it is on the east/west wall just south of the two courtyards, 39 and 48.

Figure 5 shows :
the paths of the tip of the sun's shadow at the solstices, equinoxes and for those stars that transit overhead,
the hour-lines around the pole,
the stepped arcs,
bearings around the gnomon and
radial distances from the gnomon[17] Radial distances formed part of an older table of shadow lengths.[18]

Celestial and associated phenomena influenced the layout in this area of the palace. Junctions are marked in figure 5 by small circles of radius 0.5 cubits or about 26 cms.

The following table refers to the room immediately north of court 39.

Table 1

Corners of Room to north of court 39	
Location	
SW	3rd hour-line from transit
Exit to south	Azimuth 45° and Stepped arc at 18(N),18(E) cubits from gnomon
SE	Radius 30 cubits and Winter solstice shadow
NE	Azimuth 45°
Exit to north	Radius 30 cubits
NW	Azimuth 30° and Stepped arc at 24(N) cubits from gnomon

With a 6 cubit gnomon, the line of the equator would lie above the passage linking the two courtyards and the pole would be on the more southerly of the two parallel walls. The equator coincides with the anomaly noted earlier (Figure 4). Furthermore the transit shadow of the sun at the winter solstice would fall on the end of the short passage running south from the city wall. The NW corners of both courtyards would be on a bearing of 45° from the gnomon.

[16] The two east/west walls may perhaps be linked to anomalies in Figure 4.Such walls would prevent the observer going lower for higher altitudes, and would oblige him to move nearer the gnomon.
[17] Berossus is considered to have invented the hemicycle sundial around 300 B.C. (Cousins F. W., Sundials, Redwood Press, Trowbridge, 1972, p. 72.)
[18] Hunger H. & Pingree D., Mul-Apin, Horn, 1989. p 153/4. The shadow length table is discussed in Neugebauer O., A History of Ancient Mathematical Astronomy, Vol. I, Springer-Verlag, Berlin, 1975, p 544/5, by Bremner R.W., Die Rolle der Astronomie in den Kulturen Mesopoatmiens, Symposium, Graz, 1991, pp 367/382 and by Hunger H. & Pingree D., Astral Science in Mesopotamia, Brill, 1999. pp79/82.

The proposed site seems plausible, even though having the gnomon in such a position is fraught with problems, caused by the many towers and turrets, particularly those around the palace itself. They were slender, but high and closely spaced, so that they would appear like a solid wall, if viewed obliquely.[19]

Figure 1 shows that there was an almost complete dearth of passages, near the horizon, between bearings of 6° and 22° from due east/west. To the south-east the large gateway between the Central and Principal Courts is on a bearing 12/19° from due east and could well have blocked the view to the horizon. To the south-west there is the Western Citadel, where maybe there was a similar high structure.

To check alignments at night, the observer would need to get his eye down to base level. A schematic drawing shows the palace roof as flat, but with the major north/south walls projecting above roof level.[20] In the area of the proposed observatory, the tops of all the walls were, perhaps, raised to 1.5 cubits above roof level with the gnomon 6 cubits higher still.[21] The main level at 6 cubits below the top of the gnomon would receive the shadow of the sun and, at night, the eye of someone sitting on the roof itself would be in the same plane.[22]

An additional platform, 3 cubits below the top of the gnomon, would enable the observer to measure on the horizon. He could further adjust the level of his eye by standing on a block or by kneeling[23] In the two open courtyards the observer would be able to go much lower.

Even if the observer was meant to stick to the designed paths, there would be nothing to prevent him making observations wherever he could get a sight of both the gnomon and the celestial bodies.

The moving means of the positions (Figures 4 & 6)), show that in both the west and east, the observer's path was generally close to the stepped arcs. On both sides, near the path of the sun at the summer solstice, the positions of the observer are closer to the gnomon, than indicated by the stepped curve (Figure 6). There may, perhaps, have been some sort of track marking the shadow of the sun at that extreme, preventing the observer going deeper for higher altitudes and obliging him instead to move nearer the gnomon. In such cases Ptolemy's remark about the same interval (angle) appearing *'to the observer as greater near the horizon, and less near mid-heaven'* would apply.

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[19] The turrets were closely spaced and with a width of about 6.5m. Viewed from within an angle of about 50° there would be no visible gaps between adjacent turrets.

[20] Koldewey R., The Excavations at Babylon, London, Macmillan, 1914 fig. 87. Shows cross-section through walls north of the Southern palace, with the roof of the palace shown schematically. Fig. 43 shows a birds' eye view of Southern Palace, with only some of the main walls rising above roof level.

[21] The gnomon would be 7½ cubits above the roof, which would shift the line of the sun's equinoctial shadow, from the centre of the passage, to the gnomon side of the passage wall.

[22] In the XVIII century the Jai Prakash Yantra at Jaipur similarly had complimentary sections of the two bowls cut away to allow the observer to get his eye into the plane of the bowl. (Rajawat, D.S, Jaipur's Jantar Mantar, Jaipur, date ?, pp 49/53)

[23] Analysis of the depths below the top of the gnomon suggests there was a very slight preference for certain depths: –0.5, -2.5, -3.5, -5, -6, -7, -9 and -11 cubits, but only 18% of passages were below -6 cubits.

Passages, where the depth was more than 6 cubits, are shown by heavy lines, notably in the north and due east and west of the gnomon (Figure 6). In the north-west the observer was at a significant depth over what appears to be a large area of solid brickwork, but it could have been modified without leaving a trace in the archaeological record.[24] On the east the depth was also significant in the south-east corner of court 39.

The anomaly, 5 cubits north of the gnomon, can be linked to the two courtyards and lends credence to the suggestion that those passages, well-aligned in longitude, were recorded around a gnomon in the position indicated. This is difficult to prove though, especially in the face of evidence that observers were employed by the Temple of Esagil, a long way south of the Southern Palace.[25]

Measurements

Finally we must consider what they were actually measuring..

The 656 passages were divided into six groups, according to the alignment of the two bodies in longitude, R.A or Azimuth and then whether they were observed in the east or west. For each passage the angle, in the vertical plane between the star and planet, was calculated. This angle has been termed the alignment angle and figure 7 shows how it varied with longitude. The two dashed curves are calculated values, assuming perfect alignment in longitude and with the lower of the two theoretical bodies at an altitude of 2°.

The rod, shown schematically in figure 8, would serve to check the alignment in longitude and to measure differences in latitude, assumed to be at right-angles.

Conclusions

They were using an observatory, originally laid out for the accurate determination of azimuth in linear cubits (2.5° per cubit) measured along lines parallel to the meridian. The observer would move along non-circular arcs, around the gnomon, and would be at a varying distance from the top of the gnomon. Consequently the ratio of degrees (except azimuth) per cubit would also vary.

In attempting to work in ecliptic co-ordinates, they recognised the difficulties involved. To reduce these to a minimum, they aimed to measure latitude only when they were sure the two bodies were closely aligned in longitude and this was easier close to the horizon. With close alignment in longitude, the distance between the two bodies would represent their difference in latitude.

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[24] Koldewey R & Wetzel F, Die Konigsburgen von Babylon, WVDOG54, Leipzig 1931, Die Gebaude 39 und 48 Nordlich vom Westhof. I am grateful to Helene Lambrinudi and Andreas Kindler for translations from the German.
[25] Hunger H. & Pingree D. op.cit p.139.

The layout of the area to the north of the Western Court seems to have been influenced by celestial and related phenomena. It is possible, but not proven, that the measurements could have been made there.

Appendix A – worked example

Table 2

1		**Data from Collection A**[26]		
2	Star		Planet	Difference Planet less Star or common value
3	α Virgo	Year –270/10/21	Mars	
4	172.316	Longitude °[27]	172.052	-0.264
5	-1.906	Latitude °	1.098	3.005
6		Up/Down cubits		1.5
7		Degrees Latitude per cubit		2.003
8		**Calculated Values for two bodies** Spherical trigonometry		
9		DTOG cubits - 1/Sine(row7)		28.608
10	172.197	R.A. ° (Latitude 32.5° and Obliquity of ecliptic 23.728°)	173.156	-0.958
11	1.336	Declination °	4.197	2.861
12	1.659	Altitude ° found by iteration	4.000	2.341
13	271.115	Hour-angle ° (transit 360°)	272.074	1.222
14	-0.528	Azimuth from 90°	-2.432	-1.904
15		Sun's Longitude > planet's, so passage in east & observer to west of gnomon		
16		**Calculated Positions – Observer's eye** Plane trigonometry		
17	-28.594	X cubits West (-) East (+) =DTOG x Cos (row12) x Cos (row 14)	-28.512	-0.082
18	-0.264	Y cubits South (-) North (+) =DTOG x Cos (row12) x Sine (row 14)	-1.211	-0.948
19	-0.828	Z cubits below horizontal =Row 20 x Sine(row 12)	-1.991	-1.163
20	28.596	Horizontal radius from gnomon $\sqrt{(X^2+Y^2)}$	28.538	
21		Horizontal distance between two positions – cubits $= \sqrt{(\text{Diff } X^2 + \text{Diff } Y^2)}$		0.951
22		Total cubits between two positions $= \sqrt{(\text{Diff } X^2 + \text{Diff } Y^2 + \text{Diff } Z^2)}$ Compare with recorded 1.5 cubits row 6		1.502
23		Bearing in horizontal plane from North ° = ArcTan(Diff Y/Diff X)		-85.042
24		Absolute alignment angle in vertical plane between two positions ° = ArcTan(Diff Z/row 21)		50.726

[26] A.J.Sachs & H.Hunger, Astronomical and Related Texts from Babylonia Vol I, p.351, recorded 'Night of the 19th, last part of the night, Mars was 1 ½ above α Virginis'.

[27] Longitude and Latitude of star and planet assumed unchanged over short difference in time

Appendix B

Mathematical Context in the Region.

1. A Portable Sketch from Saqqara – Pythagorean triangles and a spiral.

From Dynasty 3 (c.2600 BC), we have a sketch of an arc, which Marshall Clagett described as ‘a kind of descriptive geometry born of practical measurement…’. [28] There may be rather more to it than that.

The crucial unknown is the distance, assumed to be equal, between the Y ordinates. Clagett followed Wolff in taking it to be 28 digits or 1 Royal cubit. However, if it was actually 24 digits, the co-ordinates would be 0,98, 24,95, 48,84, 72,68 and 96,41.[29] The sketch then incorporates three Pythagorean triangles, with their long sides parallel to the X axis (Figure 9):

14, 48, 50 (7,24,25)	linking points 1 and 3,
54, 72, 90 (3,4,5)	linking points 2 and 5,[30]
30, 72, 78 (5,12,13)	linking points 1 and 4.

An Egyptian architect with Pythagorean set squares could delineate curves in integer rectangular co-ordinates, which a builder could readily follow. In this example the architect drew a rough arc on a piece of limestone, to which he added his previously calculated Y ordinates.

But what was the curve he had in mind? Points 1,3,4 & 5 lie close to a circle, but its centre (–10,-30) is well away from the vertical axis through point 1, and point 2 does not fit.

Two other possibilities are:

1.The curve is an approximate protractor for angles 15°, 30°, 45° and 67.5°.

2.The curve is part of a similar spiral to that used at Babylon, where the X coordinate is proportional to the angle below the horizontal at point 1 (see Table 3).[31] With the exception of point 3, the others are close to a ratio of 7.5° per cubit of 24 digits. This value, known as a part, or 48th of a circle, belongs ‘to an early sequence of primitive angular measures’, according to Neugebauer.[32]

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[28] Marshall Clagett, Ancient Egyptian Science, Vol. III, 1999, pp. 78/79, 109 note 68 and 462. The curve is not a single circular arc as the radius for the points 1, 2 & 3 is less than that for points 3, 4 & 5.

[29] The Egyptian small cubit contained 6 palms and 24 digits.

[30] The 3,4,5 and 5,12,13 triangles intersect at 45,79.25 and 60,68. The 11 digits just below point 2 are divided precisely into 4,3,4 digits. The triangle of 3,4,5 digits would be, in palms, ¾, 1. 1 ¼, which is similar to how it appeared in the very much later Baylonian tablet Plimpton 322 (see below).

[31] This is a similar arrangement to that at Babylon for measuring azimuth, where the ratio was 2.5° per cubit.

[32] Neugebauer. O., A History of Ancient Mathematical Astronomy, Springer-Verlag, 1975, Part Two, p.671.

The 3,4,5 triangle for points 2 and 5 fits the second alternative better than the first. (see last column in Table 3).

Table 3

Point	X	Y	Angle from Vertical at origin 0,0	Assumed Target	Difference	Angle below horizontal at point 1	Divide X by 3.2	Difference
	digits	digits	degrees	degrees	degrees	degrees	digits	degrees
1	0	98	0	0	0	0	0	0
2	24	95	14.18	15	-0.82	7.1	7.5	**-0.4**
3	48	84	**29.74**	**30**	**-0.26**	16.3	15	+1.3
4	72	68	46.64	45	+1.64	**22.6**	**22.5**	**+0.1**
5	96	41	66.87	67.5	-0.63	30.7	30	**+0.7**

2. Pythagorean Triangles and ratios of angles, including time, to linear units.

In the Old Babylonian period (ca. 1800 BC), they were well versed in Pythagorean triangles. The Ark tablet contains a value, 14430, for the necessary rope and this can be expressed as 2 x 3 x 5 x 13 x 37, where the last three factors equal the hypotenuse of a Pythagorean triangle.[33] A figure of 2405 (5 x 13 x 37) contains the hypotenuse of no less than 13 Pythagorean triangles – 5, 13, 37, 65(2), 185(2), 481(2) & 2405(4). A circle with such a radius has 108 points with integer co-ordinates, including the four cardinal points.

The more famous tablet, Plimpton 322, has 15 extant rows, each referring to a Pythagorean triangle, although some have argued that the scribe intended to complete a total of 38 rows, covering the edge and both sides of the tablet.[34] There may be good reasons why he stopped at the 15th row.

The tablet is broken and the rows are incomplete, but it is believed they would have included, in two missing columns, the short side (β) and hypotenuse (δ) of a normalised right triangle with a long side of 1. The first extant column (δ^2) is followed by expanded values b and d and finally the row number.

The 'shape of the triangles varies rather regularly ….'[35] This regularity can be improved significantly.

It is suggested that the operative part was the normalised triangle, with the expanded integer values only required to calibrate an instrument, consisting of an upright of length 1 and a horizontal bar of the same length. The horizontal bar could be moved length-wise, so that the vertical would divide it into two portions with lengths β and 1-β. There would then be two right-angled triangles, sharing a common long side of 1, with sides β, 1, δ, as defined in the tablet, and 1-β, 1, $\sqrt{(2-2\beta+\beta^2)}$ or $\sqrt{(1-2\beta+\delta^2)}$, in the ancillary triangle, which could both be scaled, as required.

[33] Finkel I. The Ark before Noah, Hodder & Stoughton, 2014, p 108. No units are actually mentioned.
[34] Brittan J.P. et al, Plimpton 322: a review and a different perspective, Arch. Hist. Exact Sci. (2011) 65 pp 519/566.
[35] Neugebauer)., The Exact Sciences in Antiquity, Dover, New York, 1969, p.38.

Scaling makes no difference to the angles in the two triangles. In the defined triangles the angles change by ca. 0.94° per row, but in the ancillary triangle it is about 1.5°, an attractive 1/60th of a quadrant.

Figures 10 and 11 plot the relationships between the angles and the short sides or the diagonals of the two triangles, several of which are closely linear up to about row 15. The ratios depend on the scaling of the triangles, which is assumed to be by a factor of 11, which is appropriate for the latitude of Babylon (32.5°). There the tangent of the celestial equator (57.5°) is 11/7. The smaller angles in the defined triangles for rows 14 and 15 are 33.3° and 31.9°, with the latter being most appropriate for latitude 31.9°. It has been argued that the tablet was from Larsa on latitude 31.2°, a little south of Babylon.

The ratios of degrees per unit of length are very close to 5° for:

The short sides of both triangles and the smaller angles in the ancillary triangles
The diagonals and the interior angles of the defined triangles.

The diagonals of the defined triangles and the angles of the ancillary triangles have a ratio of about 8°

It would be simple to change the two ratios from 5° and 8° by increasing the length of the long side from 11 to 22 or 44 respectively to give 2.5° and 2° per unit, the two ancient norms. The alternative is simply to reduce the size of the unit of measurement.

If the small angle in the ancillary triangle corresponds to the zenith distance of a star that transits overhead, the ratio of the east/west co-ordinate of the observer's eye is 6° (time to transit) per unit (see last three columns in Table 4 and figure 12). Such stars were known as *zigpu* stars at the time of mul-Apin, ca. 1000 BC.[36]

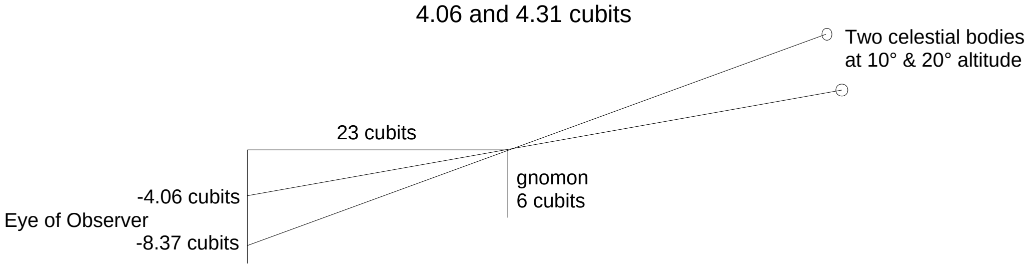
Plimpton 322 looks like a multipurpose tool for astronomers.

[36] Hunger H. & Pingree D., MUL.APIN, An Astronomical Compendium in Cuneiform, Archiv fur Orientforschung, Beiheft 24, Horn, Austria, 1989 pp 141-144. Walker C. (editor), op.cit. 1996, p.48 refers to 'A number of Late Assyrian observations and of Late Babylonian eclipse reports are timed in relation to the meridian passage of one of a group of stars known as *zigpu* stars.

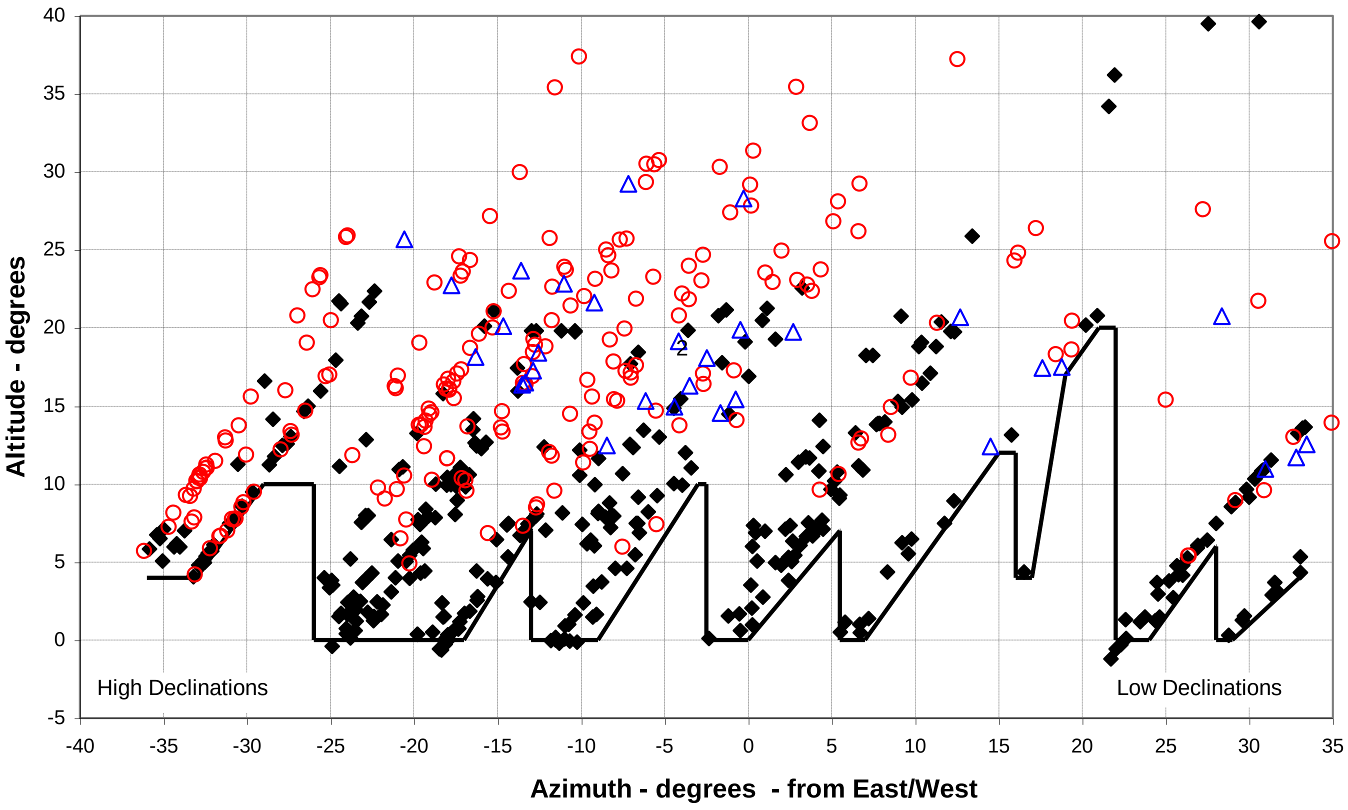
Table 4 Plimpton 322- values for rows 1 to 15, after scaling the common long side to 11 units.

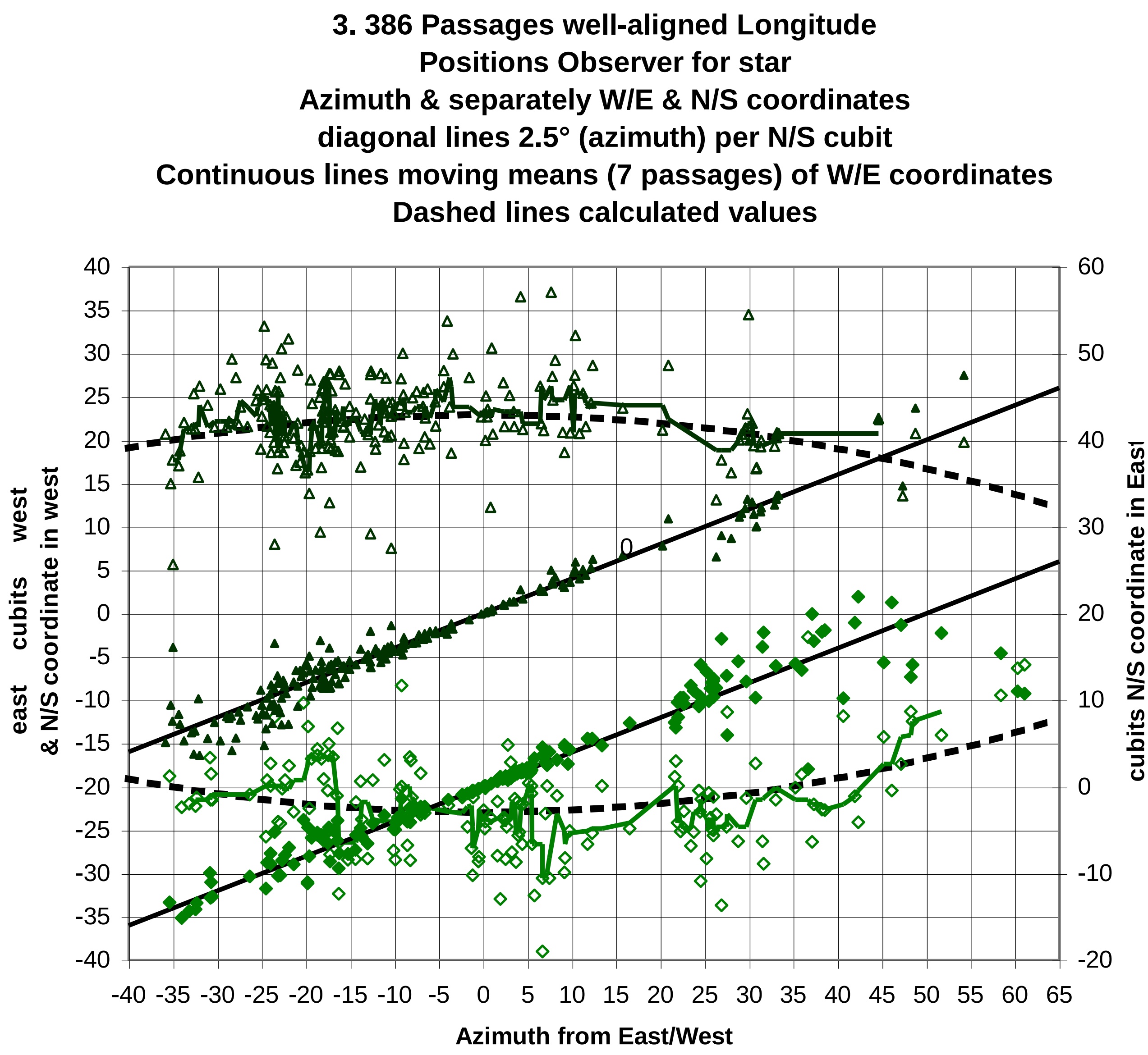
	1. Defined Triangle			2. Ancillary Triangle			Stars with Declination 32.5° On latitude 32.5°		
Row	β	δ	smaller angle	11-β	diagonal	smaller angle zenith distance	Time to transit	Position Observer's eye	
	units	units	degrees	units	units	degrees	degrees	units E/W	units N/S
1	10.91	15.49	44.76	0.09	11.00	0.48	0.57	-0.09	0.00
2	10.72	15.36	44.25	0.28	11.00	1.48	1.75	-0.28	0.00
3	10.54	15.24	43.79	0.46	11.01	2.37	2.81	-0.46	-0.01
4	10.36	15.11	43.27	0.64	11.02	3.35	3.97	-0.64	-0.01
5	9.93	14.82	42.08	1.07	11.05	5.55	6.58	-1.07	-0.03
6	9.75	14.70	41.54	1.25	11.07	6.50	7.71	-1.25	-0.05
7	9.33	14.43	40.32	1.67	11.13	8.61	10.21	-1.66	-0.08
8	9.16	14.31	39.77	1.84	11.15	9.52	11.29	-1.84	-0.10
9	8.82	14.10	38.72	2.18	11.21	11.22	13.31	-2.18	-0.14
10	8.42	13.85	37.44	2.58	11.30	13.19	15.65	-2.57	-0.19
11	8.25	13.75	36.87	2.75	11.34	14.04	16.66	-2.74	-0.22
12	7.70	13.42	34.98	3.30	11.49	16.72	19.85	-3.29	-0.31
13	7.38	13.25	33.86	3.62	11.58	18.22	21.64	-3.60	-0.37
14	7.22	13.16	33.26	3.78	11.63	18.99	22.56	-3.76	-0.40
15	6.84	12.96	31.89	4.16	11.76	20.70	24.60	-4.13	-0.48
Overall range	4.07	2.53	12.87	4.07	0.76	20.22	24.03	4.04	0.48
Ratio °/β			3.16			**4.97**			
Ratio °/δ			**5.09**			26.61			
Ratio Ancillary Angle °/ β			**7.99**			16.93			
Ratio angle °/row			0.92			**1.48**			
Ratio Altitude Per E/W unit °/unit							**5.95**		

1. Cross-section of proposed device
with observer 23 cubits from gnomon
10° intervals measure, in vertical plane,
4.06 and 4.31 cubits

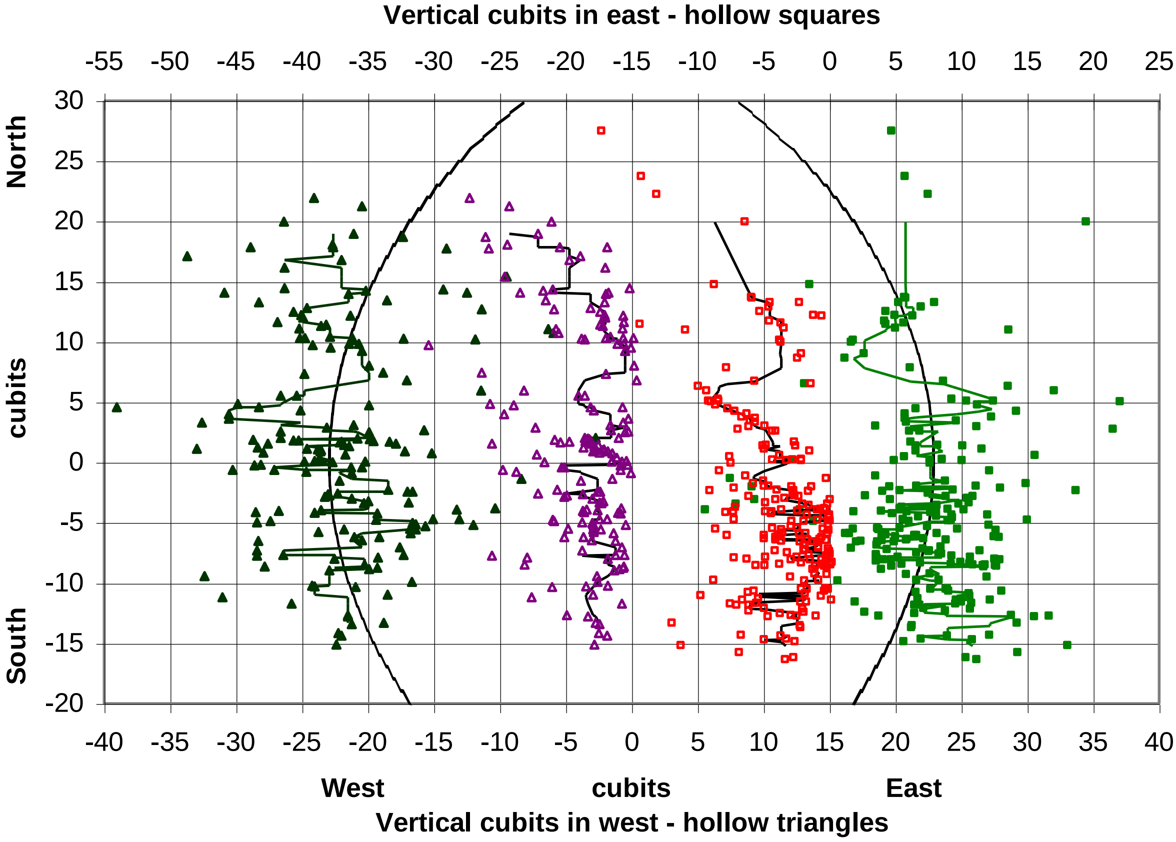


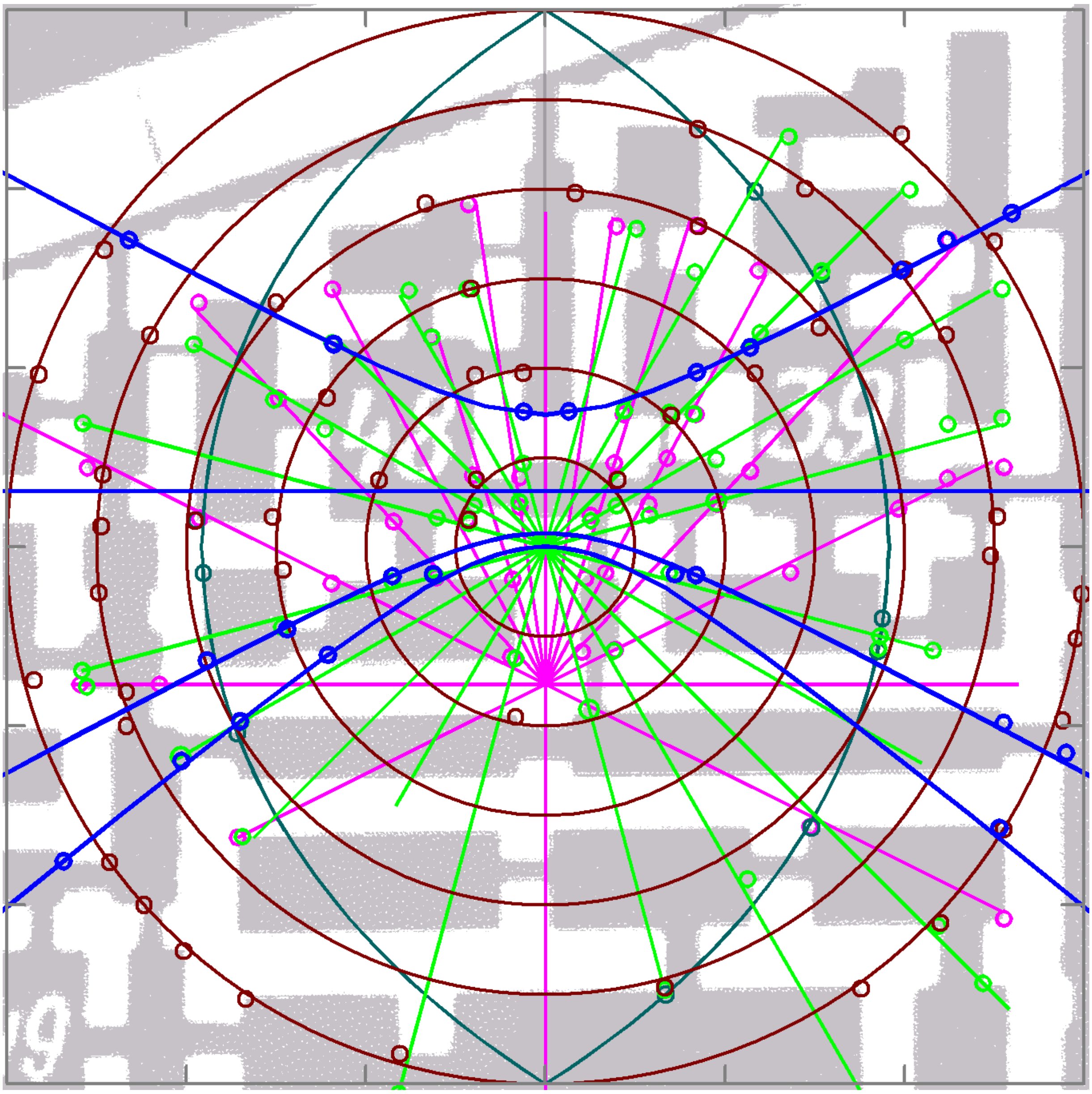
2. Positions Observer's Eye for Star
656 Passages - Calculated Distance Apart within 0.2% of Recorded Up/down Cubits
Best Alignment: Longitude - squares, R.A - circles, Azimuth - triangles
Continuous line marks lack of horizon observations (both E & W)




3. 386 Passages well-aligned Longitude
Positions Observer for star
Azimuth & separately W/E & N/S coordinates
diagonal lines 2.5° (azimuth) per N/S cubit
Continuous lines moving means (7 passages) of W/E coordinates
Dashed lines calculated values
east cubits west
& N/S coordinate in west
cubits N/S coordinate in East
Azimuth from East/West
0
40
35
30
25
20
15
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
60
50
40
30
20
10
0
-10
-20
-40 -35 -30 -25 -20 -15 -10 -5 0 5 10 15 20 25 30 35 40 45 50 55 60 65

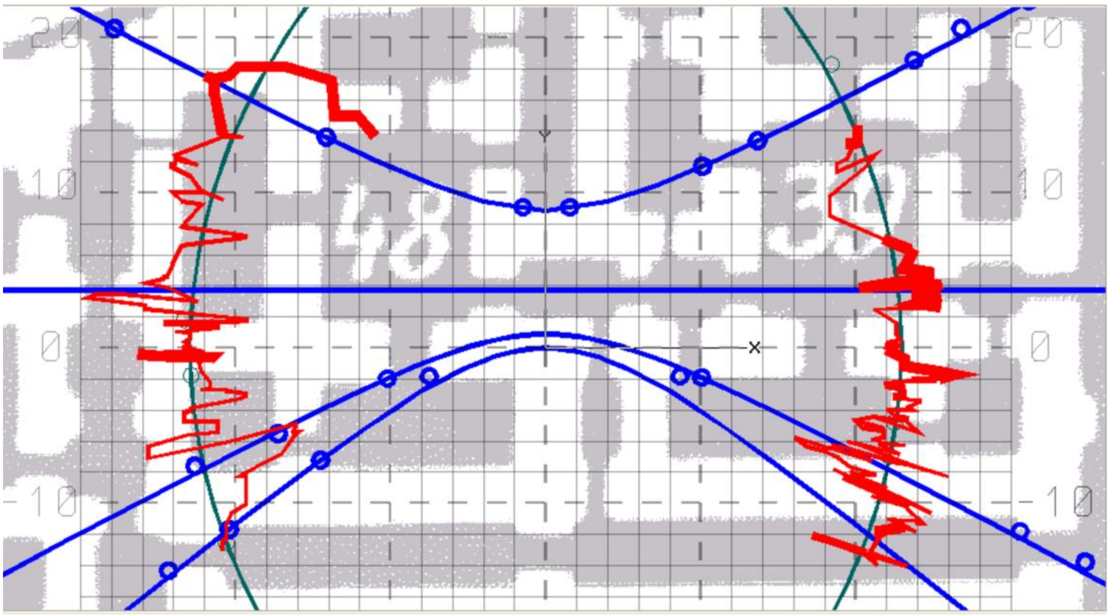
4. Passages best aligned in longitude
Observer's positions in plan & vertically
In East Observer's vertical position (top scale)
Continous lines - moving averages of 7
Dashed arcs - stepped curves
Significant anomaly ca. 5 cubits north of gnomon

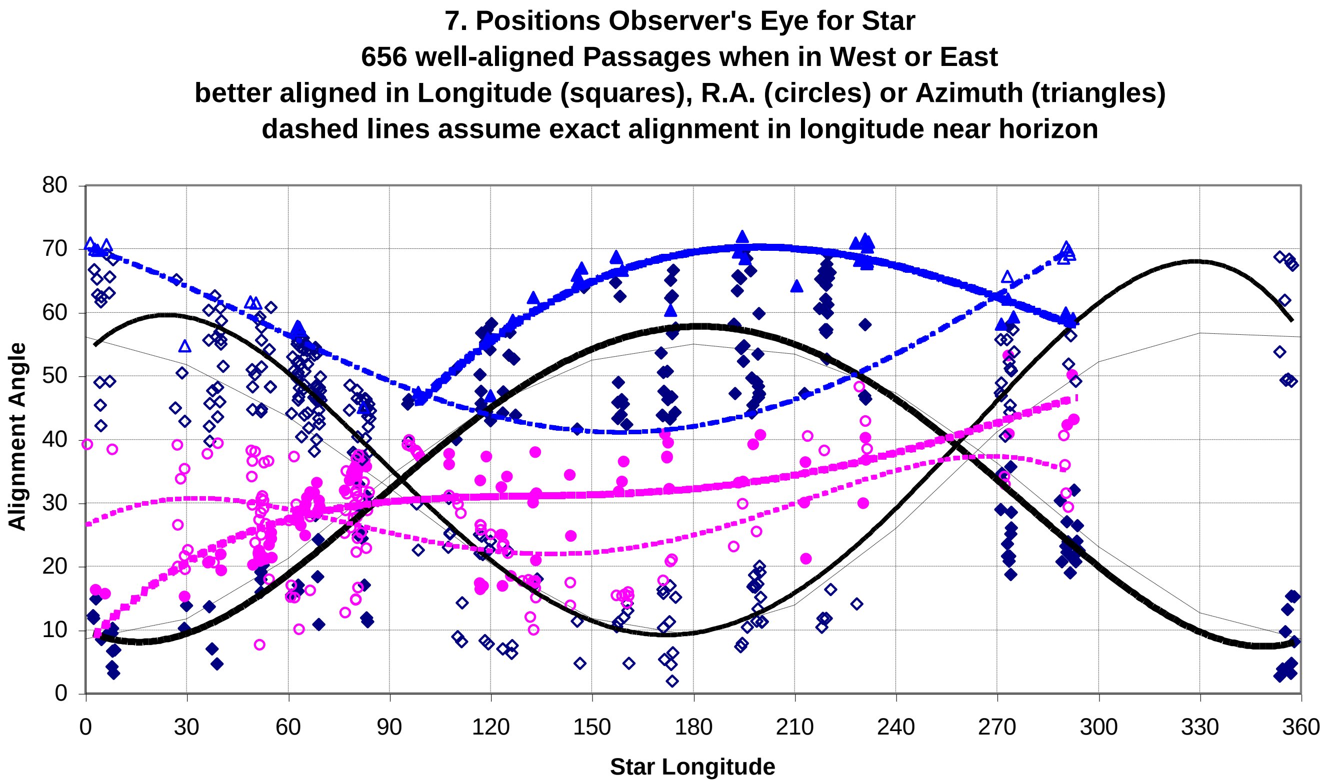




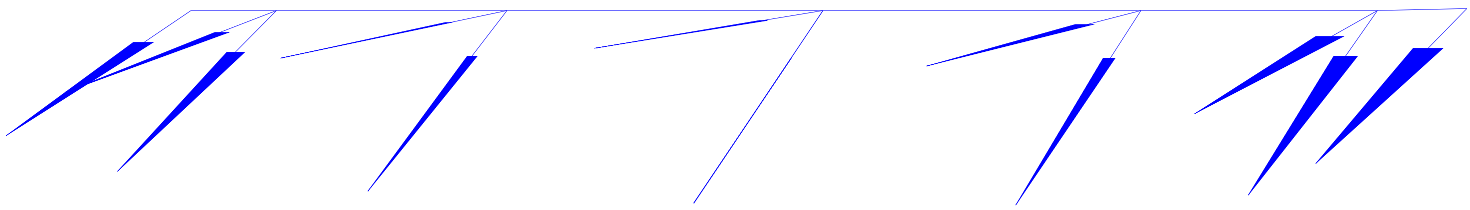
5. Plan of Babylon Palace North of Western Court,
assuming 6 cubit gnomon showing
outline square side 72 cubits – 37.4m
stepped curves for observer
concentric circles - radii from 6 to 36 cubits
lines radiating from gnomon at 15° intervals
lines radiating from pole at one hour intervals
small circles mark where lines meet wall junctions
in blue, paths of sun at equinoxes and solstices
with the equivalent path of stars that transit overhead

6. Enlargement of Figure 5, showing
calculated mean positions of observer in red
(thicker lines indicate greater depth)
square grid with sides of 2 cubits (ca. 1.04m)
in blue, paths of sun at equinoxes and solstices
with the equivalent path of stars that transit overhead



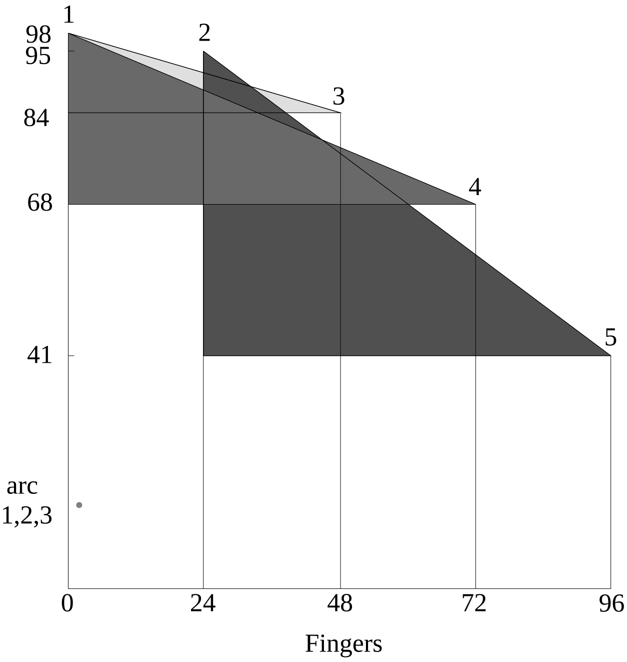

7. Positions Observer's Eye for Star
656 well-aligned Passages when in West or East
better aligned in Longitude (squares), R.A. (circles) or Azimuth (triangles)
dashed lines assume exact alignment in longitude near horizon
Alignment Angle
0
10
20
30
40
50
60
70
80
0
30
60
90
120
150
180
210
240
270
300
330
360
Star Longitude

8. Schematic Cross-section of Device in East & West
showing 4 cubit rod at 30° intervals of longitude
lower body at 2° altitude



9. Saqqara Curve with Five Points

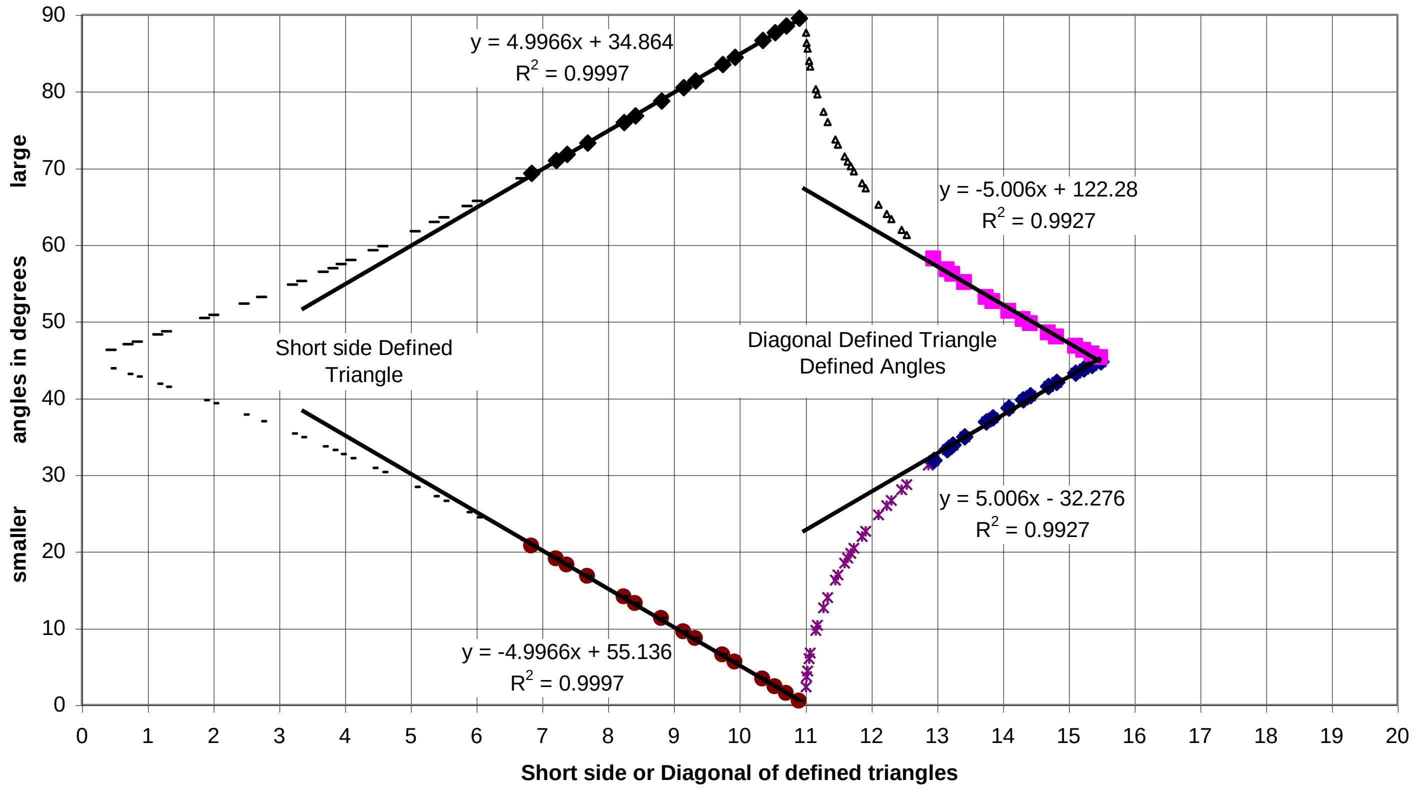
	X , Y
1	0, 98
2	24, 95
3	48, 84
4	72, 68
5	96, 41



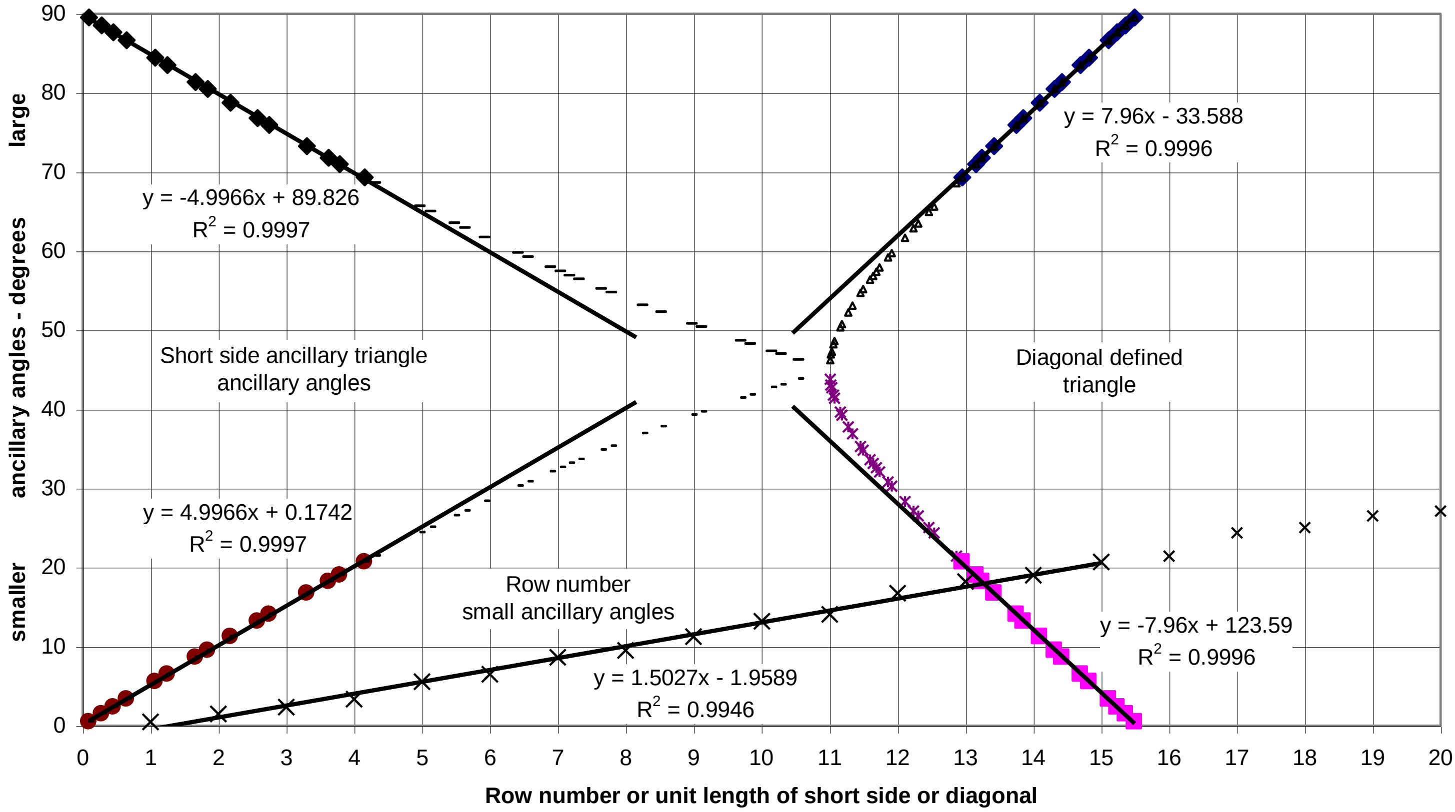
Centre arc
points 1,2,3

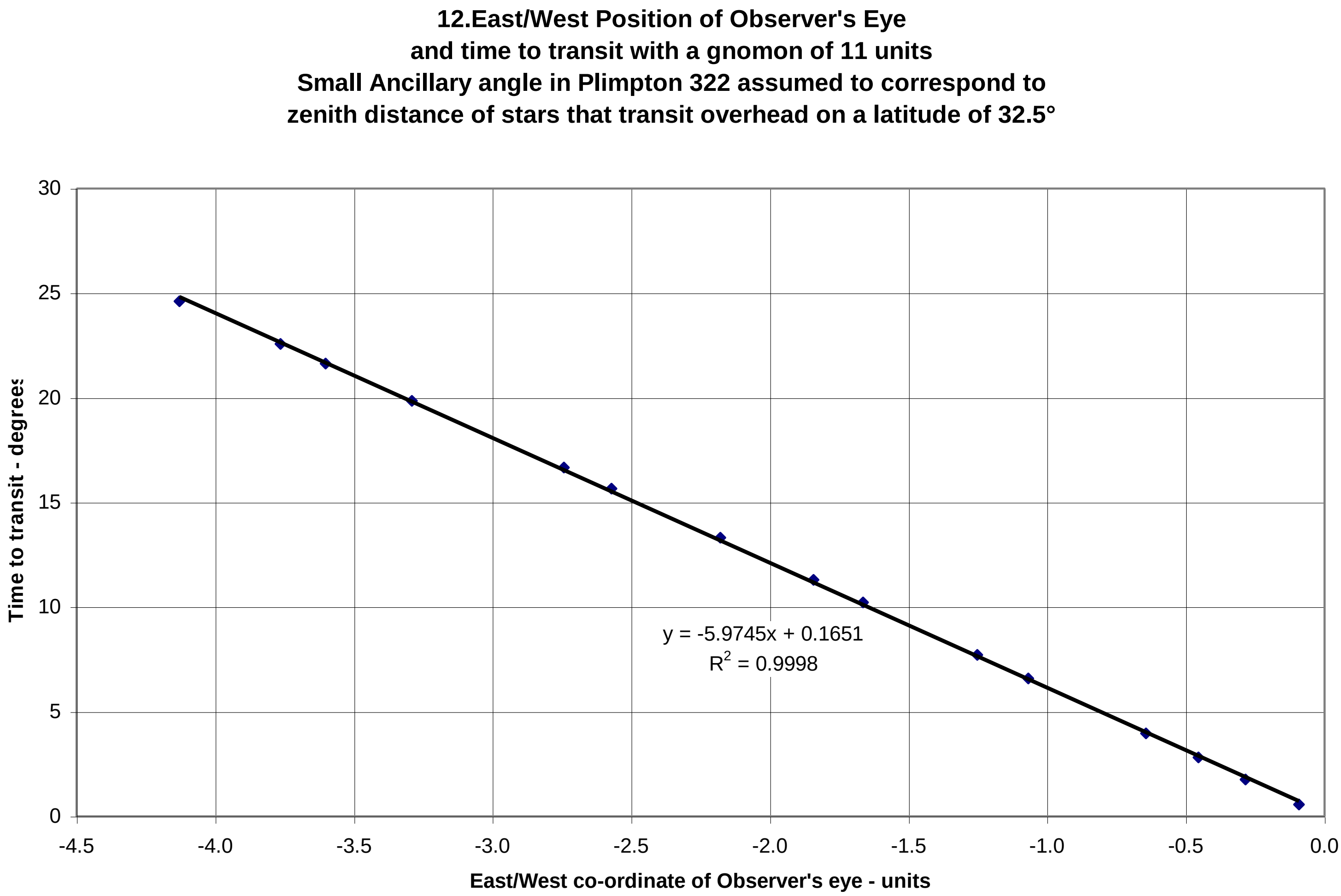
Centres Arcs
points 3,4,5
points 1,3,5

10. Plimpton 322 - Plot of short side or diagonal & angles of triangles with long side scaled to 11 units Enlarged markers & trendlines for 15 extant rows



11. Plimton 322 - Plot of short side or diagonal & angles of ancillary triangles with long side scaled to 11 units plus row number & small ancillary angles Enlarged markers & trend lines for 15 extant rows




12.East/West Position of Observer's Eye
and time to transit with a gnomon of 11 units
Small Ancillary angle in Plimpton 322 assumed to correspond to
zenith distance of stars that transit overhead on a latitude of 32.5°
Time to transit - degrees
30
25
20
15
10
5
0
-4.5
-4.0
-3.5
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0.0
y = -5.9745x + 0.1651
R2 = 0.9998
East/West co-ordinate of Observer's eye - units