Babylon: Linear Measures of Celestial Angles and an Observatory

Robert Bremner draft 2nd April,.2019

Abstract

The paper examines Babylonian records, from the 1st millennium B.C., of planets passing fixed stars and specifically their up/down differences in linear cubits. It shows they were using the top of a gnomon as a foresight around which the observer moved on non-circular arcs, where the ratio of degrees per cubit was 2.5° (azimuth). Particularly near the horizon they were able to ensure close alignment in longitude between the star and the planet. The up/down measurements were then almost identical to the distance between the two bodies, using a straight rule. Finally an area north of the Western Court of the Southern Palace is identified as a possible site of the observatory. Appendix A gives a worked example and Appendix B highlights earlier developments in the region.

This study is based on the surviving Babylonian records of planets passing fixed stars in the years from –418 to –73 BC .[1] The collection includes 1049 passages, where the up/down differences were recorded in linear units with a maximum of 6 cubits and a minimum of 1 finger (1/24th cubit). There were also 27 records of similar before and after differences. The recorded information was laconic with an up/down report for the year –418 reading simply 'Month II, night of the 9th, Mars was 4 cubits below θ Leonis'.[2]

Not all the possible combinations of fingers and cubits are represented. There were certain preferred values with rounding errors running from +/- ½ finger, for distances under 6 fingers, to +/- 6 fingers for those above 4 cubits. In percentage terms such errors reach +/- 50% with a minimum of about +/- 3%. For distances below 1 cubit, the percentage is between 7 % and 50% and above 1 cubit between 3% and 8%.

There are gaps in the longitudinal coverage of the Normal Stars and, consequently, in declination. In terms of azimuth, the gaps, near the horizon, can be seen in Figure 2.

Professor Jones concluded that the up/down cubit values were related to differences in latitude and found the mean ratio of degrees (latitude) per cubit to be about 2.3°, which lies between the two ancient norms of 2° and 2.5°.

Ptolemy, in his criticism of the data, provided clues about how the measurements were made:

In general, observations [of planets] with respect to one of the fixed stars, when taken over a comparatively great distance, involve difficult computations and an element of guesswork in the quantity measured, unless one carries them out in a manner which is thoroughly competent and knowledgeable. This is not only because the lines joining the observed stars do not always form right angles with the ecliptic, but may form an angle of any size (hence one may expect considerable error in determining the positions in latitude and longitude, due to the varying inclination [to the horizon frame of reference]); but also

[1] Jones, A., A Study of Babylonian Observations of Planets Near Normal Stars, Arch. Hist. Exact. Sci. 58 (2004) pp.475-536. I am very grateful for Professor Jones for giving me access to his Collection A and also to my son, Geoffrey, for help with the drawings and his patience.

[2] Sachs A.J. and Hunger H., Astronomical Diaries and Related Texts from Babylonia, Vienna 1988, Vol. I, p.63

because the same interval [between star and planet] appears to the observer as greater near the horizon, and less near mid-heaven;[footnote] hence, obviously, the interval in question can be measured as at one time greater, at another less than it is in reality.

[footnote] This appears to be the only reference to the effect of refraction (if that is what it is) in the Almagest, despite its obvious relevance to the observations of Mercury's greatest elongation....[3]

Clearly he considered they were thinking in terms of the ecliptic, but were also assuming lines of longitude and latitude were always at right angles.

To investigate how and what they were measuring two main assumptions were made:

1. The observer used a linear measuring rod to determine the up/down or before/after position of the planet in relation to one of the 28 so-called Normal stars. He did this by aligning one end with the star and the other point with the planet, using the top of a vertical gnomon as a foresight (Figure 1).[4] The rod may have been hand-held or fixed in a rest. For each passage, the ratio of degrees (latitude) per cubit was converted to DTOG, the distance of the observer's eye from the top of the gnomon.[5] The mean ratio of 2.3° per cubit implied that his eye was about 25 cubits from the top of the gnomon, which equates to ca. 13 metres and gives an idea of the size of the device, assuming a cubit of about 52 cms. Any errors in measurement and recording, including the not inconsiderable rounding errors, are accumulated within the DTOG value.

2. In 96% of the surviving records the difference in longitude (planet less star) was between –3.3°/+3.8°. Consequently the distance between the two bodies would be only marginally greater than their difference in latitude. By repeated iterations 656 passages (63% of the surviving records) were found where the distance between the two bodies was within 0.2% of the recorded up/down distance in cubits.[6] The margin of 0.2% is small but it equates to a 3.6° difference in the before/after positions, if a rectangular co-ordinate system was used, as Ptolemy indicated.[7]

Professor Jones calculated the ecliptic co-ordinates of the outer planets for midnight and those of the inner planets about 4 hours, either before or after midnight.[8] No adjustments were made either to these celestial co-ordinates or for refraction.

Appendix A has a worked example of the calculations for one passage.

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[3] Toomer G.J., Ptolemy's Almagest, Duckworth, London, 1984, p.121.
[4] The main justification for such an arrangement is that it brings the measuring scale close to the observer. It is not essential as the observer could be at the centre of the device with the scales about 13m away, but in that case it is hard to accept the 'measurements' as much more than estimates.
[5] DTOG, distance from top of gnomon, equals 1/sine(ratio degrees per cubit)
[6] Varying the altitude of the two bodies changes the linear distance between them. Out of the 128 passages with up/down distances of 4 fingers or less and rounding errors greater than 12.5%, only 29 had the distance apart within 0.2% of the recorded value.
[7] Using plane trigonometry, Cosine (3.6°) = 0.998.
[8] Jones A., op cit. p.481 gives UT 21 for the outer planets and either 17 or 1 UT for the inner planets.

Before and After Alignment

There has been considerable discussion about their ability to measure in ecliptic co-ordinates.[9] Of the 656 passages 59% were most closely aligned in longitude particularly at lower altitudes. In 78% of these passages, the longitude difference (planet less star) was less than 1°, compared with 63% of all surviving records. Other passages were better aligned in R.A. (34%) or even azimuth (7%) (Figure 2). This confirms Professor Jones's conclusion that they were thinking in ecliptic co-ordinates, but, it now appears, their alignments, in longitude, were closer at lower altitudes. The mean altitude of the stars for the passages, best aligned in longitude, was 7.5° and, for the others, 16.6°.

Positions of the Observer's Eye

The passages, best aligned in longitude, were sorted to the order of the star's azimuth, in the west and the east. The relationships between azimuth and, separately, the north/south and east/west cubit co-ordinates of the eye of the observer are shown in figure 3.

Surprisingly the relationship between azimuth and the north/south co-ordinates is very close to linear, with each cubit corresponding to 2.5° of azimuth, which implies that the paths of the observer were neither circular arcs around the gnomon nor straight lines.[10] Instead those paths must have been stepped arcs.[11] This provides a good indication of the intended paths of the observer in the east and west.[12] However, in practice, within 20° of due east/west, the divergence from a straight line is less than 1 cubit and could well have been ignored, if a straight line was more acceptable.

The observer's position in the vertical is generally within 6 cubits of the top of the gnomon, but drops to 10 or 11 cubits in places (Figure 4). There is a notable anomaly about 5 cubits north of the gnomon, where, particularly in the east, the observer's eye drops down to about 10 cubits. This anomaly also marks a sharp fall in the number of passages, when the observer is between 5 and 10 cubits north of the gnomon.

From the foregoing we can deduce that there was a structure around the gnomon which facilitated observations where the observer's eye was within 6 cubits of the top of the gnomon.

There are other aspects brought out by the moving mean lines in figure 4.[13]

[9] Hunger H. & Pingree. D. Astral Sciences in Mesopotamia, Brill, 1999, p.269.

[10] This also confirms that they were thinking in terms of 2.5° per cubit.

[11] I am most grateful to P. Starkey, a neighbour and mathematician, for providing the modern polar equation for such curves: $r \sin \theta = (Y_{max} .2/\pi)\ \theta$, where r is radius and θ the angle in radians. Spiral curves may have been used in Egypt at an early date (see Appendix B). A similar linear relationship, but closer to 2.6° (azimuth) per cubit, was found for those passages best-aligned in R.A., indicating that for those passages the observer was slightly closer to the gnomon. As they were also higher, it implies that the observer's path, in cross-section, was like a steep-sided bowl.

[12] A target ratio of 2.5° per cubit implies the distance to the top of the gnomon on the east/west line was about 23 cubits (1/22.9 = Tan 2.5°). From there it is simple to calculate thirty-six cubit steps, each of 2.5°, to the north and south. With due north/south being at 0,36/0,-36 and due east/west at ca. 23,0/-23,0. The intermediate positions at 45° are +/-18, +/-18. With 36 steps the sum of each successive hypotenuse totals 44.6 cubits, so along that path each cubit averages about 2°. In practice the steps may have been irregular and larger than the 1 cubit assumed. If used to measure altitude, rather than azimuth, such a curve would resemble the recumbent crescent moon, a common motif in Mesopotamia, but an impossible position for the moon in practice.

[13] Moving means help to smooth out erratic data, but depend on how the data was sorted. In figure 4 it was in order of N/S cubits, but in figure 6 in order of azimuth.

A Possible 'Observatory'

To visualise the observatory, we might think of a 6 cubit gnomon standing above the flat roof of a building with the much lower areas corresponding either to the ground outside or to interior open courtyards. In the Southern Palace at Babylon, there are many such courtyards, but there is an area north of the Western Court of particular interest. Before the whole of the area had been excavated, a part to the north-east was described as follows:

The houses of this part of the palace are remarkable for the strength of their walls and the admirable regularity with which they are laid out. Court 38 is reached by a passage-way from the Principal Court, the latter through a hall, as in the case of 25, 26 and 27, opens with three doors on to court 38. Between the doors, pillars project from the walls and correspond with others on the opposite side. They must have served as piers to support arches for the ceiling, although it is difficult to make out clearly what was the object of this structure.

The roof of this area of the palace was evidently intended to support more weight than usual. It may appear improbable that an observatory would be rectangular, but we can perhaps think of it as being like graph paper. Today we use Mercator charts, with rectilinear lines of longitude and latitude, and also Ordnance Survey maps with a rectangular grid. It is a question of balancing the pros and cons of such arrangements.

There are circles in the sky which produce straight lines, aligned with the cardinal directions, on the ground. Firstly there is the meridian. Secondly a prime purpose of an observatory would have been the measurement of time both at night and during the day. In a horizontal sundial the hour-line for 6 hours to transit runs due west/east through the pole. Thirdly the shadow of the sun, at the equinoxes, runs due west/east just north of a gnomon.[14] We thus have three perfectly straight lines – the meridian, the hour-line for six hours to transit and the shadow of the sun at the equinoxes – and we have already noted that the stepped arcs run sensibly due north/south within 20° (8 cubits) of due east/west. Together these lines form a near rectangular outline for observations.

Just south-east of court 48 is a short length of wall of abnormal width (1.8m), which is aligned with a passage leading from the northern wall of the palace. None of the other similar passages, running due south, from the oblique northern wall, is so short.[15] The wide wall and the short passage may perhaps have marked the meridian.

The short thick wall links two substantial east/west walls, about 5m apart; one just south of house 48 and the other north of the transverse corridor.[16]

The stepped curves, with each north/south cubit corresponding to 2.5° of azimuth, would fit within the north/south width of this part of the palace, with the gnomon about midway

[14] With a gnomon of 1 cubit, on a latitude of 32.5°, the pole would be 1.57 cubits to the south and the equator 0.637 to the north, with the distance between them being 2.207 cubits.

[15] The oblique northern wall of the palace is stepped, both vertically and horizontally, and is inclined about 17° from east/west. The 17° of azimuth matches that quoted for the limits of the path of Anu in Walker C. (editor), Astronomy before the Telescope, British Museum Press, 1996, p.48. It corresponds to the rising/setting of stars with a declination of +/- 14.3°, which is close to the 15°, for the Path of Anu, quoted in Hunger H. and Pingree D., Astral Sciences in Mesopotamia, Brill, Leiden, 1999, p.61.

[16] The two east/west walls may perhaps be linked to anomalies in Figure 4. Such walls would prevent the observer going lower for higher altitudes, and would oblige him to move nearer the gnomon.

between the two east/west walls. However, as we will see, there are reasons to believe it was perhaps ca. 2m further north. In figures 5 & 6 it is on the east/west wall just south of the two courtyards, 39 and 48.

Figure 5 shows :
the paths of the tip of the sun's shadow at the solstices, equinoxes and for those stars that transit overhead,
the hour-lines around the pole,
the stepped arcs,
bearings around the gnomon and
radial distances from the gnomon[17] Radial distances formed part of an older table of shadow lengths.[18]

Celestial and associated phenomena influenced the layout in this area of the palace. Junctions are marked in figure 5 by small circles of radius 0.5 cubits or about 26 cms.

The following table refers to the room immediately north of court 39.

Table 1

Corners of Room to north of court 39	
Location	
SW	3rd hour-line from transit
Exit to south	Azimuth 45° and Stepped arc at 18(N),18(E) cubits from gnomon
SE	Radius 30 cubits and Winter solstice shadow
NE	Azimuth 45°
Exit to north	Radius 30 cubits
NW	Azimuth 30° and Stepped arc at 24(N) cubits from gnomon

With a 6 cubit gnomon, the line of the equator would lie above the passage linking the two courtyards and the pole would be on the more southerly of the two parallel walls. The equator coincides with the anomaly noted earlier (Figure 4). Furthermore the transit shadow of the sun at the winter solstice would fall on the end of the short passage running south from the city wall. The NW corners of both courtyards would be on a bearing of 45° from the gnomon.

The proposed site seems plausible, even though having the gnomon in such a position is fraught with problems, caused by the many towers and turrets, particularly those around the palace itself. They were slender, but high and closely spaced, so that they would appear like a solid wall, if viewed obliquely.[19]

Figure 2 shows that there was an almost complete dearth of passages, near the horizon, between bearings of 6° and 22° from due east/west. To the south-east the large gateway between the Central and Principal Courts is on a bearing 12/19° from due east and could

[17] Berossus is considered to have invented the hemicycle sundial around 300 B.C. (Cousins F. W., Sundials, Redwood Press, Trowbridge, 1972, p. 72.)
[18] Hunger H. & Pingree D., Mul-Apin, Horn, 1989. p 153/4. The shadow length table is discussed in Neugebauer O., A History of Ancient Mathematical Astronomy, Vol. I, Springer-Verlag, Berlin, 1975, p 544/5, by Bremner R.W., Die Rolle der Astronomie in den Kulturen Mesopoatmiens, Symposium, Graz, 1991, pp 367/382 and by Hunger H. & Pingree D., Astral Science in Mesopotamia, Brill, 1999. pp 79/82. See also Appendix B, pages 18/21.
[19] The turrets were closely spaced and with a width of about 6.5m. Viewed from within an angle of about 50° there would be no visible gaps between adjacent turrets.

well have blocked the view to the horizon. To the south-west there is the Western Citadel, where maybe there was a similar high structure.

To check alignments at night, the observer would need to get his eye down to base level. A schematic drawing shows the palace roof as flat, but with the major north/south walls projecting above roof level.[20] In the area of the proposed observatory, the tops of all the walls were, perhaps, raised to 1.5 cubits above roof level with the gnomon 6 cubits higher still.[21] The main level at 6 cubits below the top of the gnomon would receive the shadow of the sun and, at night, the eye of someone sitting on the roof itself would be in the same plane.[22]

An additional platform, 3 cubits below the top of the gnomon, would enable the observer to measure on the horizon. He could further adjust the level of his eye by standing on a block or by kneeling[23] In the two open courtyards the observer would be able to go much lower.

Even if the observer was meant to stick to the designed paths, there would be nothing to prevent him making observations wherever he could get a sight of both the gnomon and the celestial bodies.

The moving means of the positions (Figures 4 & 6)), show that in both the west and east, the observer's path was generally close to the stepped arcs. On both sides, near the path of the sun at the summer solstice, the positions of the observer are closer to the gnomon, than indicated by the stepped curve (Figure 6). There may, perhaps, have been some sort of track marking the shadow of the sun at that extreme, preventing the observer going deeper for higher altitudes and obliging him instead to move nearer the gnomon. In such cases Ptolemy's remark about the same interval (angle*)* appearing *'to the observer as greater near the horizon, and less near mid-heaven*' would apply.

Passages, where the depth was more than 6 cubits, are shown by heavy lines, notably in the north and due east and west of the gnomon (Figure 6). In the north-west the observer was at a significant depth over what appears to be a large area of solid brickwork, but it could have been modified without leaving a trace in the archaeological record.[24] On the east the depth was also significant in the south-east corner of court 39.

The anomaly, 5 cubits north of the gnomon, can be linked to the two courtyards and lends credence to the suggestion that those passages, well-aligned in longitude, were recorded around a gnomon in the position indicated. This is difficult to prove though, especially in

[20] Koldewey R., The Excavations at Babylon, London, Macmillan, 1914 fig. 87. Shows cross-section through walls north of the Southern palace, with the roof of the palace shown schematically. Fig. 43 shows a birds' eye view of Southern Palace, with only some of the main walls rising above roof level.
[21] The gnomon would be 7½ cubits above the roof, which would shift the line of the sun's equinoctial shadow, from the centre of the passage, to the gnomon side of the passage wall.
[22] In the XVIII century the Jai Prakash Yantra at Jaipur similarly had complimentary sections of the two bowls cut away to allow the observer to get his eye into the plane of the bowl. (Rajawat, D.S, Jaipur's Jantar Mantar, Jaipur, date ?, pp 49/53)
[23] Analysis of the depths below the top of the gnomon suggests there was a very slight preference for certain depths: –0.5, -2.5, -3.5, -5, -6, -7, -9 and -11 cubits, but only 18% of passages were below -6 cubits.
[24] Koldewey R & Wetzel F, Die Konigsburgen von Babylon, WVDOG54, Leipzig 1931, Die Gebaude 39 und 48 Nordlich vom Westhof. I am grateful to Helene Lambrinudi and Andreas Kindler for translations from the German.

the face of evidence that observers were employed by the Temple of Esagil, a long way south of the Southern Palace.[25]

Measurements

Finally we must consider what they were actually measuring..

The 656 passages were divided into six groups, according to the alignment of the two bodies in longitude, R.A or Azimuth and then whether they were observed in the east or west. For each passage the angle, in the vertical plane between the star and planet, was calculated. This angle has been termed the alignment angle and figure 7 shows how it varied with longitude. The two dashed curves are calculated values, assuming perfect alignment in longitude and with the lower of the two theoretical bodies at an altitude of 2°.

The rod, shown schematically in figure 8, would serve to check the alignment in longitude and to measure differences in latitude, assumed to be at right-angles.

Conclusions

They were using an observatory, originally laid out for the accurate determination of azimuth in linear cubits (2.5° per cubit) measured along lines parallel to the meridian. The observer would move along non-circular arcs, around the gnomon, and would be at a varying distance from the top of the gnomon. Consequently the ratio of degrees (except azimuth) per cubit would also vary.

In attempting to work in ecliptic co-ordinates, they recognised the difficulties involved. To reduce these to a minimum, they aimed to measure latitude only when they were sure the two bodies were closely aligned in longitude and this was easier close to the horizon. With close alignment in longitude, the distance between the two bodies would represent their difference in latitude.

The layout of the area to the north of the Western Court seems to have been influenced by celestial and related phenomena. It is possible, but not proven, that the measurements could have been made there.

[25] Hunger H. & Pingree D. op.cit p.139.

Appendix A – worked example

Table 2

1		**Data from Collection A**[26]		
2	Star		Planet	Difference Planet less Star or common value
3	α Virgo	Year –270/10/21	Mars	
4	172.316	Longitude °[27]	172.052	-0.264
5	-1.906	Latitude °	1.098	3.005
6		Up/Down cubits		1.5
7		Degrees Latitude per cubit		2.003
8		**Calculated Values for two bodies** Spherical trigonometry		
9		DTOG cubits - 1/Sine(row7)		28.608
10	172.197	R.A. ° (Latitude 32.5° and Obliquity of ecliptic 23.728°)	173.156	-0.958
11	1.336	Declination °	4.197	2.861
12	1.659	Altitude ° found by iteration	4.000	2.341
13	271.115	Hour-angle ° (transit 360°)	272.074	1.222
14	-0.528	Azimuth from 90°	-2.432	-1.904
15		Sun's Longitude > planet's, so passage in east & observer to west of gnomon		
16		**Calculated Positions – Observer's eye** Plane trigonometry		
17	-28.594	X cubits West (-) East (+) =DTOG x Cos (row12) x Cos (row 14)	-28.512	-0.082
18	-0.264	Y cubits South (-) North (+) =DTOG x Cos (row12) x Sine (row 14)	-1.211	-0.948
19	-0.828	Z cubits below horizontal =Row 20 x Sine(row 12)	-1.991	-1.163
20	28.596	Horizontal radius from gnomon $\sqrt{(X^2+Y^2)}$	28.538	
21		Horizontal distance between two positions – cubits = $\sqrt{(\text{Diff } X^2 + \text{Diff } Y^2)}$		0.951
22		Total cubits between two positions = $\sqrt{(\text{Diff } X^2 + \text{Diff } Y^2 + \text{Diff } Z^2)}$ Compare with recorded 1.5 cubits row 6		1.502
23		Bearing in horizontal plane from North ° = ArcTan(Diff Y/Diff X)		-85.042
24		Absolute alignment angle in vertical plane between two positions ° = ArcTan(Diff Z/row 21)		50.726

[26] A.J.Sachs & H.Hunger, Astronomical and Related Texts from Babylonia Vol I, p.351, recorded 'Night of the 19th, last part of the night, Mars was 1 ½ above α Virginis'.
[27] Longitude and Latitude of star and planet assumed unchanged over short difference in time

Appendix B - Earlier Developments in the Region.

1. Horizon Alignments.

The supposed temple at Tell es-Sawwan is an early example, from the middle of the 6th millennium BC, of a building oriented about 45° from the cardinal points.[28] Later at Teleilat Ghassul (level IV) in Palestine there is a remarkable wall painting of an eight-pointed star.from about –4000.[29]

Other bearings are evident at Nabta Playa and Eridu.

Nabta Playa.

At Nabta Playa there are alignments of megaliths radiating around a central point. Their bearings are in three bands A (26/31°), B (117/122°) and C (127/131°).[30] In turn these can be divided into narrower ranges, but here we will look at the positions of the individual stones as, with a fixed central point, it only takes one marker to define an alignment.

For each megalith, the differences in longitude (converted to great circle degrees) and latitude, from the central point, were divided by a factor.[31] . A unit of .00194°, corresponding to c. 215 metres, gave significant results for bands A and B.[32] Of the 17 positions no less than 8 had longitudes (expressed in linear units) equating to either whole or half units. Of the radii from the central point 8 equated to either whole or half units. This cannot be accidental. It would appear that they were determining positions by any two of the following: the radius from the centre and the easterly or northern component from the centre. In other words any two sides of a right-angled triangle.

Band C does not fit this analysis, which is not surprising as Malville et al concluded it was 'problematic because of migration of the stones.[33] However one stone (C5) is still of interest, as its unit co-ordinates are 2.7 (S) and 3.6 (E) and with a radius of 4.5 units from the centre. The alignment corresponds to the hypotenuse of a Pythagorean triangle with sides in the ratio 3, 4 & 5. In this case the unit would be 193.5m.(10% smaller than mentioned above). There is the possibility that one of the attractions for the placing of the central point (A) was its position relative to C5, which was described as a 'dispersed cluster of blocks' with a large original size of 'about 2.0 x 1.5 x 0.3 m.'[34]

Bands A and B are largely confined to two 3.4° segments, between bearings determined by the ratio ½, the tangent of 26.6° and the sine of 30°, measured from due North (A) or

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[28] Edwards I.E.S et al, The Cambridge Ancient History, Vol.1, Part 1, 1980, p.274, Fig. 21.
[29] Edwards, op. cit, Vol. IV, p.522 and plate 14c.
[30] Malville J.M. et al, Astronomy of Nabta Playa, in Holbrook J. et al, African Cultural Astronomy, Springer 2008, p.137.
[31] Brophy T.G and Rosen P.A, Satellite Imagery Measures of the Astronomically Aligned Megaliths at Nabta Playa, Mediterranean Archaeology and Archaeometry, 2005, Vol.5, No.1, pp15-24, Table 1.
[32] The calculation is based on a great circle degree of 111 km. According to Petrie (Encyclopaedia Britannica 1951), the Egyptians had a khet (100 cubits) with a length of 52.37m. 4 khets would be 210m. Subdivisions smaller than a half, were probably tenths rather that quarters or thirds. There is some indication that the unit length rose from about 211m in –4400 to 218m in –3600. If we assume that the three A positions A1, A2 & A3 were all intended to be 4 units north and 2 units east of the centre, the units would range from .00185 to .00195°.
[33] Malville op. cit, p.139
[34] Wendorf F. and Malville J.M, The Megalithic Alignments in Wendorf F. and Schild R, The Archaeology of Nabta Playa, 2001, p.494.

due East (B). The two segments are 90° apart. The bearing of the rising sun at the winter solstice would have been 26.1°, south of due east, in -4700

From the table below, we can see that the rising of Sirius, the brightest star in the sky, would have aligned with the stones in the B band from about –4700 to –3700, but with a gap from –4200 of nearly 400 years.[35] An adjustment of nearly 2° was then needed.

Similarly the rising of Arcturus would have matched the megaliths in the A band from –4450 to –3600, but with the largest gap from –4275 to –4100. To put these dates in perspective, it is thought the Egyptian Civil calendar with 365 days in the year was established around –4500.[36]

Table 3 Individual Megaliths at Nabta Playa

Ref	Size	Position		Difference		Linear measures			
		Lat.	Long	Lat	Long	Lat	Long	Radius	Year BC
	Cu.m.	Degrees	degrees	Degrees x 100	Gt. Circle degrees x 100	units	Units	units	
Centre A		22.5080	30.7257						Arcturus
A2	3.7	22.5157	30.7298	0.77	0.38	**4.0**	**2.0**	4.4	4450
A3	0.7	22.5155	30.7297	0.75	0.37	3.9	1.9	4.3	4430
A1	2.9	22.5158	30.7299	0.78	0.39	**4.0**	**2.0**	**4.5**	4400
A0	0.4	22.5136	30.7288	0.56	0.29	2.9	**1.5**	3.2	4275
A4	1.4	22.5149	30.7297	0.69	0.37	3.6	1.9	**4.0**	4100
AX	0.4	22.5164	30.7306	0.84	0.45	4.3	2.3	4.9	4075
A5	1.4	22.5131	30.7288	0.51	0.29	2.6	**1.5**	**3.0**	3920
A6	?	22.5135	30.7291	0.55	0.31	2.8	1.6	3.3	3850
A7	0.5	22.5131	30.7289	0.51	0.30	2.6	**1.5**	**3.0**	3800
A8	1.0	22.5127	30.7287	0.47	0.28	2.4	1.4	2.8	3720
A9	1.0	22.5121	30.7284	0.41	0.25	2.1	1.3	**2.5**	3600
									Sirius
B7	0.5?	22.5065	30.7283	-0.15	0.24	-0.8	1.2	**1.5**	4700
B6	0.1	22.5063	30.7288	-0.17	0.29	-0.9	**1.5**	1.7	4460
B5	?	22.5061	30.7293	-0.19	0.33	**-1.0**	1.7	**2.0**	4200
B3	5.2	22.5059	30.7300	-0.21	0.40	-1.1	**2.0**	2.3	3820
B1	?	22.5058	30.7303	-0.22	0.42	-1.1	2.2	**2.5**	3750
B4	?	22.5060	30.7299	-0.20	0.39	**-1.0**	**2.0**	2.3	3700
C5	0.9?	22.5027	30.7333	-0.53	0.70	-2.7	3.6	**4.5**	

This analysis indicates that 12 of the 17 megaliths in bands A and B were placed in three short periods of greater activity: –4450/ –4400 (4), -4100/-4075 (2)–3850/-3700 (6). Only two were placed in the 300 years from –4400 to -4100 (exclusive), which matches the three centuries, when the lowest number of samples were found for radiocarbon dating (Figure 8B). We can perhaps see this period as being one of low human activity in the area and is consistent with the megaliths in the A & B bands being placed individually to point to the rising of Arcturus or Sirius.

The megaliths would also align with other less bright stars. For example Sirius and α Centaurus rose at the same point on the horizon around –4400 and thereafter markers which had served previously for Sirius would serve for α Centaurus, as it moved lower in the sky.

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[35] Star data from SkyMap Lite 2005.

[36] Wells R.A. in Walker C. (Editor), Astronomy before the Telescope, British Museum, 1996, p.34

The distance from the central point would vary as they sought integer values for linear measurements of any two of the radius, latitude or longitude, to determine the precise position

In general they seem to have been less tolerant of imprecision in the case of Arcturus than Sirius. Consequently there are more alignments for the former, possibly because the slow movement northwards of the rising of Sirius was already well known. Unlike Sirius, the rising of Arcturus was moving southwards, which may have attracted closer attention.[37] The first four alignments for Arcturus are near 26.6°, with a tangent of 0.5. The difference in bearing for these four was less than one degree, which suggests an aim for high precision.

Eridu.

At Eridu not all the many levels of temple construction were perfectly rectangular and the early walls varied significantly in bearing.[38] At Napta Playa the lines of stones, radiating around a centre, point solely to the eastern horizon, but at Eridu the walls can be seen as aligned between opposite points on the western and eastern horizons (Table 4).

Table 4 – Walls at Eridu

Level		Walls					Stars			
		SE	NW	NE	SW		α CMa	α Cen	α Lyr	κ Ori
		Bearings - degrees				Year	Longitude/Horizon Azimuth-degrees			
18		30/210[39]	30/210							
17		30/210	29/209	126/306	127/307	-5100	8/**127**			
16		30/210	30/210	126/306	126/306	-4900	10/**126**		189/25	
15		35/215	39/219	130/310	130/310	-4700	13/125	155/121	192/26	354/**129**
11		37/217	37/217	127/307	127/307	-4250[40]		161/124	199/29	0/**126**
9		37/217	37/217	127/307	127/307	-3750		167/**127**	205/32	7/122
8		40/220	41/221	132/312	133/311	-3000		177/**132**	210/36	
7		40/220	40/220	131/311	131/311		See footnote 31			
6		53/233	53/233	143/323	143/323	-2700		181/226	220/**323**	
Level 6 excluded		Corresponding Declinations Degrees								
18/7	rising	48/41	49/41	-30/-35	-30/-36					
18/7	setting	-48/-41	-49/-41	**30/35**	**30/36**		Zigpu stars in bold (on left)			

We can distinguish four distinct groupings:

1. In each of the first three levels, 18/16, there is at least one wall oriented 30°/210°. This suggests a subdivision of the horizon into 30° segments, with the two middle segments, totalling 60° in the east and west, corresponding to slightly more than the annual range of the sun at the horizon.[41] The 30°/210° alignment would complete the 30° segments and would mark the centres of the two bands, which the sun does not

[37] The rising of Arcturus moved 5° southwards in 850 years and of Sirius 4.7° northwards in 1000 years

[38] The alignments were taken from Edwards I.E.W., Gadd C.J., Hammond N.G.L. (Editors), The Cambridge Ancient History, CUP, 1980, Figures 24 & 25, pp 335 & 338. Figure 24 shows levels 18 to 8 and although small has the advantage of having just one indication of north for all levels. In figure 25, the other two levels, 7 & 6, each have their own north pointer. In this analysis level 7 with an indicated date around –3100 would be out of sequence with level 8.

[39] Three of the walls are aligned 30/210°, while one, the most northerly, is about 29/209°

[40] α Cma and α Cen would have had the same declination c. –4400, which falls between levels 15 and 11.

[41] In the middle of the 6 millennium BC, with the obliquity of the ecliptic 24.2°, the theoretical range would be 29.4° either side of due east/west.

reach and which are not circumpolar. The divisions between the major 60° segments lie either side of an alignment 60/240° or 120/300°.

2 In levels 11 and 9 the buildings are more closely rectangular and oriented in accordance with the angles in the simplest Pythagorean triangle, with sides in the ratio 3,4,5.[42] One wall at level 17 is similarly aligned.[43] The same angles are also evident in the last level (6) but transposed. Six of the nine identified levels had walls in this or the previous group.

3 Excluding levels 18 and 6, the remaining seven have at least one wall on a bearing of 126/132° in the east and 306/312° in the west. These two ranges correspond to objects with complementary declinations of –30/-35° and +30/+35°, either rising in the east or setting in the west. The latter range would include what were later termed zigpu stars, which transit overhead and ideally had a declination of 30.5° at Eridu.[44] The former range would, at different times, have included two of the brightest stars as at Nabta Playa.

Of the five brightest stars Canopus (α Car) and Arcturus would have been too low or too high, leaving Sirius, α Cen and Vega (α Lyr). The brightest star, Sirius, would have risen on a bearing of 127° in –5100 and 126° in –4900, when it would have been opposite, in longitude, to Vega and so six months apart. As Sirius rose, Vega was 21° above the western horizon. Later Sirius became too high, but α Cen would have been in range (levels 11, 9 & 8). This leaves a gap between levels 15 and 11, which could have been filled by a star of Orion, such as Saiph (k Ori), which, although not particularly bright, is part of a very obvious constellation and was also opposite the sun at the autumn equinox. An alternative would have been the brighter Rigel (β Ori)

With levels 17 and 16 two hundred years apart, we might estimate the date of level 18 as about –5300. Overall the range would be from then until level 6 in –2700. Postgate gives a range from c.-5000 to c.-3000.[45] Bienkowski and Millard give a span of 'at least 1500 years from 5500 BC or earlier'.[46] The dates suggested here, although not coincident, are similar to those indicated by these two authorities. We can probably have the greatest confidence in those for levels 17 and 16, associated with the rising of Sirius, levels 11 or 9 and 8 associated with the rising of Rigel Kentaurus and level 6, associated with the setting of Vega.[47]

Egyptian 5-pointed star.

In the coffin lid tables (see below) the epagomenal stars are grouped together, but we should not rule out the possibility that at some earlier stage a single day was inserted into the calendar every 72 days.

The star hieroglyph with five spokes, implying the division of a circle into 72° segments, is known from the earliest Dynasties.[48] It is not the easiest form to draw, so there must have been a good reason for its adoption.[49] It is shown with one spoke vertical and the others on either side at angles of 72° and 144°. Table 5 gives details of five stars which, at

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42 The angles are 36.9° and 53.1°

43 It is possible that there may have been some lack of differentiation between the various levels.

44 Hunger H and Pingree D, MUL-APIN, An Astronomical Compendium in Cuneiform, Archiv fur Orientforschung, Horn, Austria,1989, pp 141/4.

45 Postgate J.N., Early Mesopotamia, Routledge, London, 1996, p.25 caption to figure 2:2.

46 Bienkowski P and Millard A. Dictionary of the Ancient Near East, British Museum, London, 2000, p.107.

47 Other than for level 6, Vega seems to have been consistently mis-aligned by about 4/9°.

48 Petrie H, Egyptian Hieroglyphs of the First and Second Dynasties, Quaritch, London, 1927.

Abydos around –3900, would have, almost simultaneously, been on the horizon. The two rising stars, λ Tel and 110 Her, are not particularly bright.

Table 5 Calculated for –3900 at Abydos (Lat. 26.2°)

Star	Magnitude	R.A.	Declination	Horizon Azimuth	Diff Azimuth
α UMi	1.86	322	58	343	71
γ Gem	1.93	19	0	271	72
α Car	-0.62	66	-58	199	71
λ Tel	4.85	184	-34	128	71
110 Her	4.19	220	32	54	75

It would not have taken long to realise that α UMi spent about one fifth of a day below the horizon and was separated from β UMa by a similar length of time. These two northern stars would have facilitated the visual subdivision of the area around the pole into five equal segments.

Therefore a plausible alternative justification for the hieroglyph would be that the spokes are separated by 72° in time. α UMi with a declination of 58.7° would rise and set 36° (time) from lower transit and 144° from upper transit. It would be 72° below the horizon between setting and rising, which would be 36° apart in azimuth.

α Umi would have had such a declination around –3900 and at the same time it and β UMa would have been 72° apart (R.A.). Other stars with about the same declination would have been γ Dra and one of those in the Corona Borealis constellation. As α Umi set, α Car was also setting, which provides additional support for the five-pointed star being related to the rotation of α Umi around the pole.[50] The suggested date of –3900 is commensurate with the –4500 given by Wells for the determination of the length of the year as 365 days.[51]

Table 6 Calculated for –3900 at Abydos (Lat. 26.2°)

Star	Magnitude	R.A	Difference	Declination	Horizon Azimuth	Time to nearest transit	Long approx
		degrees	degrees	degrees	degrees	degrees	degrees
α UMi	1.86	322	83	58	17	35	7
β UMa	2.34	34	72	62	8	17	57
α2 CVn	2.84	94	60	64	n.a.	n.a	
α CrB	2.21	167	72	56	21	42	140
γ Dra	2.24	240	73	61	12	5	186

A few decades later α UMi would spend 69° below the horizon, which would match the 70 days spent in the duat, which traditionally is associated with the time that Sirius (α CMa) is too near the sun to be visible. Maybe there was more than one manifestation of the 70 days in the duat.

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[49] Roaf. M, Cultural Atlas of Mesopotamia, Equinox, Oxford, 1990, p.70. shows a pictographic sign for a star with eight spokes around –3100.

[50] At this time α Umi and α Car would have set 143° apart in azimuth.

[51] Wells R.A., op.cit. p.34.

By this time the five-pointed star might have come to represent the daily passage of time, around the pole. The rising and setting of a Umi (R.A.322.5) could have served as a control, with the other four stars being a Uma (25), e Uma (105), y CrB (171) and e Dra (253.5). The successive differences in R.A (in brackets). range from 62° to 83°, so would not have been at all precise.

In the Pyramid texts, the word for hours is determined by three stars.[52] Sticking with α Umi, the other two could have been β Cva (86) and η Her (202.5). The differences in R.A would be 116.5°, 120° and 123.5° and, if correct, would indicate much greater precision. This is speculative, but seeks to explain how the measurement of time could have reached the high level of precision built into Kafre's and later pyramids (see below).

Alignment of Mastabas at Saqqara

The northerly alignments of all but one of the long sides of the mastabas of the 1st Dynasty at Saqqara are in one of two groups 330/341° and 355/358°.[53] The first is roughly parallel to the Nile, which along this stretch flows towards 335°.

With this relationship to the river, it would not have gone unnoticed that around –2920, when α UMi was at upper transit, the setting of Corona Borealis was aligned with the river (Table 7). We see that constellation as a crown, but then it might have been likened to a bowl or the sign N41/42, a 'well full of water'.[54] On setting its 'rim' would have been level with the horizon on a bearing between 331/341°, matching the first of the two groups of mastaba alignments.[55] At the same time α CMa would have been 2° below the horizon and about to rise.

Table 7 Data for –2920 on a latitude of 30°, α Umi at upper transit

Star	Magnitude	R.A.	Decl	Horizon Azimuth	Altitude
		degrees	degrees	degrees	degrees
ι CrB	4.98	190	52	335	0.1
ε CrB	4.14	188	49	331	-3.5
γ CrB	3.8	183	49	331	-5.0
α CrB	2.21	180	51	333	-4.9
β CrB	3.65	179	53	337	-3.0
θ CrB	4.06	181	55	341	0.1
α UMi	1.86	330	63	N/a	57.0
α CMa	-1.44	47	-22	115	-2.0

52 Clagett M, op.cit. Vol.II p.49. He presumes that this was linked to the Civil Calendar with 12 months in three seasons.
53 One mastaba is aligned 10° west of north, about midway between the two groups.
54 Gardiner Sir A., Egyptian Grammar, Oxford University Press, 3rd Edition, 1969, p. 492.
55 Over the years in question, precession would not have played a significant role in the spread of the mastaba alignments.

2. A Portable Sketch from Saqqara – Pythagorean triangles and a spiral.

From Dynasty 3 (c.2600 BC), we have a sketch of an arc, which Marshall Clagett described as 'a kind of descriptive geometry born of practical measurement…'. [56] There may be rather more to it than that.

The crucial unknown is the distance, assumed to be equal, between the Y ordinates. Clagett followed Wolff in taking it to be 28 digits or 1 Royal cubit. However, if it was actually 24 digits, the co-ordinates would be 0,98, 24,95, 48,84, 72,68 and 96,41.[57] The sketch then incorporates three Pythagorean triangles, with their long sides parallel to the X axis (Figure 9):

14, 48, 50 (7,24,25) linking points 1 and 3,
54, 72, 90 (3,4,5) linking points 2 and 5,[58]
30, 72, 78 (5,12,13) linking points 1 and 4.

An Egyptian architect with Pythagorean set squares could delineate curves in integer rectangular co-ordinates, which a builder could readily follow. In this example the architect drew a rough arc on a piece of limestone, to which he added his previously calculated Y ordinates.

But what was the curve he had in mind? Points 1,3,4 & 5 lie close to a circle, but its centre (–10,-30) is well away from the vertical axis through point 1, and point 2 does not fit.

Two other possibilities are:

1.The curve is an approximate protractor for angles 15°, 30°, 45° and 67.5°.

2.The curve is part of a similar spiral to that used at Babylon, where the X co-ordinate is proportional to the angle below the horizontal at point 1 (see Table 8).[59] With the exception of point 3, the others are close to a ratio of 7.5° per cubit of 24 digits. This value, known as a part, or 48th of a circle, belongs 'to an early sequence of primitive angular measures', according to Neugebauer.[60]

The 3,4,5 triangle for points 2 and 5 fits the second alternative better than the first. (see last column in Table 8).

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[56] Marshall Clagett, Ancient Egyptian Science, Vol. III, 1999, pp. 78/79, 109 note 68 and 462. The curve is not a single circular arc as the radius for the points 1, 2 & 3 is less than that for points 3, 4 & 5.
[57] The Egyptian short cubit contained 6 palms and 24 digits.
[58] The 3,4,5 and 5,12,13 triangles intersect at 45,79.25 and 60,68. The 11 digits just below point 2 are divided precisely into 4,3,4 digits. The triangle of 3,4,5 digits would be, in palms, ¾, 1. 1 ¼, which is similar to how it appeared in the very much later Baylonian tablet Plimpton 322 (see below).
[59] This is a similar arrangement to that at Babylon for measuring azimuth, where the ratio was 2.5° per cubit.
[60] Neugebauer. O., A History of Ancient Mathematical Astronomy, Springer-Verlag, 1975, Part Two, p.671.

Table 8 Analysis of Five Points in Sketch

Point	X	Y	Angle from Vertical at origin 0,0	Assumed Target	Difference	Angle below horizontal at point 1	Divide X by 3.2	Difference
	digits	digits	Degrees	degrees	degrees	degrees	digits	degrees
1	0	98	0	0	0	0	0	0
2	24	95	14.18	15	-0.82	7.1	7.5	**-0.4**
3	48	84	**29.74**	**30**	**-0.26**	16.3	15	+1.3
4	72	68	46.64	45	+1.64	**22.6**	**22.5**	**+0.1**
5	96	41	66.87	67.5	-0.63	30.7	30	**+0.7**

3. Standard Pyramids and 'Hour' Standards.

The great pyramids of Giza were built in the 4th Dynasty and by the 6th there was a de facto standard shape. Six such pyramids were built in the years after 2500. They are now in poor condition, but originally the sides and corners, respectively, had slopes of 53.13° and 43.314°.[61] At the equinoxes, the sun will reach such altitudes 1.5 and 2.5 hours from transit. The precise times depend on the geographic latitude as shown in Table 9, where for comparison Khafre's pyramid at Giza, with a slightly greater slope, is also included.[62]

Table 9 Pyramid Characteristics

Pyramid	Dynasty and approx. date BC[63]	Location	Latitude[64] Best Estimates	Sides		Corners		Difference Time to Transit
				Slope	Time to Transit	Slope	Time to Transit	From 1 hour
			Degrees	Degrees	Hours	Degrees	Hours	Seconds
Khafre	4 – 2556	Giza	29.976	53.167	**1.499**	43.352	**2.505**	+23
Userkaf	5 – 2492	Saqqara	29.873	53.130	1.513	43.314	2.514	+4
Neferirkare[65]	5 – 2473	Abusir	29.895	53.130	1.511	43.314	2.513	+7
Teti	6 – 2343	Saqqara	29.877	53.130	1.513	43.314	2.514	+5
Pepi I	6 – 2319	S.Saqqara	29.855	53.130	1.515	43.314	2.515	**+1**
Merenre	6 – 2285	S.Saqqara	29.851	53.130	1.515	43.314	2.515	**+1**
Pepi II	6 – 2276	S.Saqqara	29.839	53.130	1.516	43.314	2.516	**-1**

Although this equinoctial time standard was built into the pyramids, it must have been developed using a skeletal version with a line-of-sight to the apex from inside when 1.5 hours from transit. Twice a year the sun would indicate such standard times, with the 1-hour difference between them being very exact, particularly for the later pyramids at South Saqqara.[66] The difference would be exactly one hour, with the sun precisely on zero declination, on a latitude of 29.846°, which lies in the Wadi Tafla between the pyramids of

[61] 53.13° corresponds to the middle angle in a Pythagorean triangle with sides in the ratio 3,4,5.
[62] Lehner, M, The Complete Pyramids, Thames & Hudson, London , 1997, p.17 summarises the characteristics of the pyramids and on page 161 gives a small plan of the Pepi II complex.
[63] These dates are based on Shaw I. & Nicholson P., Dictionary of Ancient Egypt, British Museum, 1997, p.310, with a conventional 2 years added to the accession dates to arrive at the dates of the layout. I am grateful to Dennis Rawlins for pointing out the inconsistent dates in the earlier version of this table in my letter to the BAA Journal, Vol. 127.1, 2017.
[64] Based on The Times Atlas of the World, Comprehensive Edition, London, 1992 and Google Earth.
[65] Pyramid incomplete.
[66] The characteristics of Khafre's pyramid suggest that the main objectives were the two individual times to transit and not the difference between them. At the summer solstice this time difference is reduced from 60 to about 45 minutes.

Merenre and Pepi II. This area would have been off the plateau and so unsuitable, otherwise those two pyramids might have been even closer to this ideal latitude.

In the Pyramid Texts, on the walls of 5th and 6th dynasty pyramids, Utterance 251 includes 'O you who are over the hours…..' and Utterance 320 'The King has cleared the night, the King has despatched the hours….'.[67] 'In both passages the word for hours (*wnwt*) is determined by three stars, suggesting to us that the most primitive meaning of "hours" was "nighttime hours".'[68] The more precise measurement of time by the stars was evidently established by the 5th Dynasty (2500/2350 B.C.), but they would then have needed to identify stars close enough to the equator (Table 10).

[67] R.O. Faulkner, The Ancient Egyptian Pyramid Texts, OUP, 1969. The earliest surviving example is in the pyramid of Unas (2373 BC), but no single pyramid contains the whole text.
[68] M. Clagett, opo.cit,Vol. II, American Philosophical Society, Philadelphia, 1995, p 49.

Table 10 – Stars with magnitude <5 near equator[69]

Star	RA	Magnitude							
	-2300		-2500	-2450	-2400	-2350	-2300	-2250	-2200
			Declination - Arc Minutes						
δ3 Taurus	0.6	4.29							-2
ε Taurus	0.6	3.53		-2	15				
11 Orion	18.7	4.65						-12	4
15 Orion	19.8	4.81				-11	4		
134 Taurus	29.8	4.89			-5	10			
28 Monoceros	65.4	4.68						-7	0
TYC 4857 2151 1	72.1	3.91					-12	-7	-2
27 Hydra	86.8	4.80							-12
υ2 Hydra	98.4	4.60						12	9
y Hydra	145.2	2.99				5	-9		
β 1 Scorpius	184.0	2.56				9	-7		
ω2 Scorpius	184.1	4.31	-4						
ω1 Scorpius	184.1	3.93	6	-11					
ν Scorpius	185.6	4.00				9	-8		
ψ Ophiuchus	188.2	4.48	-11						
χ Ophiuchus	189.2	4.18				17	0		
TYC 6221 904 1	192.8	4.91				4	-12		
η Ophiuchus	200.0	2.43				2	-14		
o Serpens	208.1	4.26					7	-8	
ξ Scutum	219.0	4.66					7	-6	
α Scutum	222.1	3.85					4	-9	
η Scutum	227.9	4.83					7	-14	
12 Aquila	229.1	4.02		9	-3				
λ Aquila	230.4	3.43				5	-6		
ι Aquila	238.8	4.36						6	-3
η Andromeda	320.3	4.40	-3	10					
λ Aries	334.3	4.79			-6	8			
α Aries	336.3	2.01			-14	1			
ξ Aries	352.1	4.00							-8
Number of Stars			4	4	5	11	13	9	8
Closest pair or 0			-3/+6	-2/+9	-3/+15	-11/+1	0	-6/+6	0
Balanced pair			-4/+6	-11/+10	-15/+14	-11/+10	-7/+7	-6/+6	-3/+4

The final four pyramids had sides of 150 cubits and were originally 100 cubits high. Consequently, the shadow of the equator would run in a straight line, 57.4 cubits north of the apex.[70] In the last and most southerly, that of Pepi II, we can 'see the plan of the standard pyramid complex in its final and most developed form'.[71] Uniquely it had an added girdle, 6.5 metres (12.4 cubits) in width, which increased the sides of the base from 150 to 174.8 cubits.[72]

In relation to the main pyramid, those of the wives were positioned using Pythagorean triangles (Table 11 and Figure 9a).

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69 Data from SkyMap Lite 2005

70 The calculation is 100 x Tangent (geographic latitude). For the most northerly, that of Userkaf, the distance would be 57.44 cubits.

71 I.E.S. Edwards, The Pyramids of Egypt, Penguin, 1993, p.181.

72 Edwards, op.cit. p.188. The girdle, an addition to the original plan, may have been required for reinforcement. We do not know its height, but the width was 6.5m or 12.4 cubits. On the small plan the side measured 172.5 cubits, compared with the calculated value of 174.8 cubits, a difference of ca. 1.3%. This gives a rough idea of the precision of the measurements.

Table 11 Pyramids of wives

Pyramid	Centre Measured on plan		Pythagorean Triangle & (scaling)	Calculated centre	
	West	North		West	North
	cubits	cubits		cubits	cubits
Iput II	160.8	119.1	3,4,5 (40)	160	120
Neith	66.4	155.8	5,12,13 (13)	65	156
Wedjbeten	79.4	152.4	8,15,17 (10)	-80	-150

On the other hand, some distances in the Pyramid complex appear to be based on a standard, related to the distance of the equinoctial line from the centre (Table 12).

Table 12 (First six rows are measured on plan, the others are calculated*)*

	Distance cubits	Divisor	Unit of Measurement cubits
Eastern Wall of Mortuary Temple to centre pyramid	259.8	9	28.9
Girdle side	172.5	6	28.7
Satellite Pyramid N	114.7	4	28.7
Satellite Pyramid E	72	2½	28.8
Diagonal Open court	58.1	2	29.0
Diagonal of Iput II	56.7	2	28.4
Equinoctial shadow line	57.36	2	28.7
Distance on equator between 35° & 50° & between 40° & 50°	56.7 40.7	2 √2	28.3 28.8
Distance from meridian on equator (time to transit)	316.7 (70°) 199.7 (60°) 137.4 (50°) 115.3 (45°) 96.7 (40°) 66.6 (30°) 42.0 (20°)	11 7 4¾ 4 3⅓ 2⅓ 1½	28.8 28.5 28.9 28.8 29.0 28.5 28.0

This suggests that a unit of about 28.8 cubits was used for some aspects of the layout, with 12.5 of these units being 360 cubits. Today we might think of it as 90/ π and a circle with this radius would have a circumference of 180 cubits with a ratio of 2° per cubit, one of the ancient norms.[73] Then it might have been seen as determining the equatorial shadow line (2x) and as a useful linear unit for the measurement of time from the meridian along the equator (Table 12).[74]

The equatorial line coincides with the northern wall of the mortuary temple, but the hour standard, identified in table 5, cannot be accommodated, as already mentioned. On the other hand, before the addition of the girdle around the base of the pyramid, an hour, from 35° and 50° after transit, would fit neatly within the open area immediately north of the sanctuary. In that position it would serve for objects in the western sky, using the apex as a

[73] 90/π equals 28.8, if π is taken to be 25/8. Intriguingly at just over 72° from the meridian, the distance is 360 cubits and 5th Dynasty representations of stars show them with five points.

[74] The later Shadow Clock, described in the Cenotaph of Seti 1 is different, as it appears to use hours of 60 minutes. (Clagett op. cit. pp.463/470 has a translation).

foresight. In the narrow gap between the pyramid and the enclosure wall in the west, a star could only be observed close to 35° before transit (ignoring the girdle).[75] In the pyramid of Pepi II, the girdle would reduce the level area along the equatorial line. It is suggested that, to overcome this setback, they opted for a short hour of 10° or 40 minutes. For 40° and 50° from transit the observer would be, respectively, 96.7 and 137.4 cubits from the meridian, with the difference being close to an average of 4 cubits per 1° of time or 1 cubit per minute.[76] A short hour of 10° appears a little later in the diagonal star tables on coffin lids.[77]

The sanctuary would restrict observations of stars above the equator, but to the south the absolute limit would be –18° declination on the meridian.[78] Away from the meridian such a body could only be observed from outside the enclosure wall. Within it and north of the pyramid the declination would be around –12°, which, crossing the meridian 90 cubits north of the apex, avoids the girdle and allows observation along the length of the enclosure wall.

Significantly the causeway for Khafre's pyramid had an azimuth, directed at the rising of a body with a more precisely defined declination of –11.8°.[79] The sun would have such a declination two months from the winter solstice and would delimit a season of the four months with the 120 shortest days.[80] A calendar for an Egyptian year of three seasons could thus be kept in step with the sun, with the other two seasons being either side of the summer solstice.

At night, four bright stars were in the band between 0° and –12° of declination (Table 13). Sirius itself was too low, but the Sothis constellation included her head-dress, so δ Monoceros, with a similar R.A., is taken as the exemplary star. One of the 36 ten-day decans is 'Red One of Khenett', identified as the red α Scorpio (Antares, Rival of Mars). Between it and δ Monoceros there were 136 days and 13 decans, which are sufficiently correlated to justify the identifications.

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[75] In round numbers stars on the equator could not be observed within 35° of the meridian, mimicking the 70 days passed in the 'duat'. See Clagett op.cit p. 364/5, referring to the Book of Nut.

[76] The ratio would be exactly 1 cubit per minute, on average, between 39.5° and 49.5° from transit, with the distances from the meridian being 95.0 and 135.0 cubits.

[77] Wells R.A., op.cit., pp 37/8. The earliest of these tables date from the 9th Dynasty, soon after the reign of Pepi II.

[78] The declination of Sirius would only have risen to –18° by 1425 BC.

[79] Nell, E. and Ruggles C.,The Orientations of the Giza Pyramids and associated structures, University of Leicester, version 2 – 15th March 2013, p.37, Table 12.

[80] The sun's R.A. being 208° & 332° with a difference of 124°.

Table 13 Bright Stars with declinations between 0° & -12°

Star[81]	Magnitude	Equatorial Co-ordinates -2300		Julian Day Re-based	Diff R.A.	T class 10 day decans [82]	
		R.A.	Decl.	days	degrees	No.	Re-based
α Taurus	0.75	11.7	-1.1	-179	-176	24?	-18?
y Orion	1.64	26.3	-7.5	-164	-161	26?	-16?
α Orion	0	33.0	-4.1	-157	-155	27?	-15?
δ Monoceros	4.15	53.8	-4.3	-136	-134	29	-13
α Scorpio	0.88	187.8	-7.8	0	0	6	0

These stars are close to the equator, where we have seen that the distance between 40° and 50° from transit is just over 40 cubits and for bodies with a declination of –12° it would be 38 cubits between 30° and 40°.[83] In both cases it would average about 1 cubit per minute. The Egyptians were able to measure time (months, days and hours) rather better than is usually acknowledged.

4. Coffin Lid Tables in Egypt.

Two centuries after the building of the last standard pyramid we have the first coffin lids with astronomical tables. These tables list 36 decan stars, at 10-day intervals, plus 5 epagomenal days, in accordance with the Egyptian calendar. We have seen above that they were using an hour of 60 minutes, but the girdle added to the pyramid of Pepi II, may have forced them to employ a shorter hour of 40 minutes. They would then have had to rework earlier schemes and, on this basis, we suggest dating the surviving coffin lid tables to about –2250.

The majority of the coffin lids of known provenance come from Asyut on a latitude of 27.23°, which has certain interesting properties. The sun at the solstices would rise 27.16° from due east, which is almost identical to the height of the pole.[84] Less obviously the azimuth, swept by the sun at the two extremes, would be 153° and 207°, if measured from rising to 270°, which closely matches the time of the sun above the horizon, 154° and 206°. A change of 1° azimuth corresponded, on average, to 1° time above the horizon. They had the means to measure time for celestial bodies with declinations between +/-30°.

[81] Star data from StarMap Lite 2005. The interest in an hour standard may have been stimulated by α Taurus and γ Orion, which in 3300 BC were precisely that time apart.

[82] S. Symons, S, A Star's Year in J.M. Steele (editor), Calendars and Years, Oxbow, 2007. p.8 (Table 5). Decans 24 and 25 refer to the Arm [of Orion]. In the K class (Table 6) the difference, between Sothis and the Red One, is 14, not 13, decans.

[83] For comparison, the distance for the original hour standard (Table 5) was 41 cubits for 60 minutes. The distances along the equatorial line being 48.3 and 89.2 cubits for altitudes of 53.13° and 43.314°.

[84] Assuming 23.95° for the obliquity of the ecliptic.

Table 14 Asyut - Latitude 27.23° Rising Azimuth and Time above Horizon

Rising Azimuth from North	Declination	Azimuth swept to 270°	Time above horizon
Degrees	degrees	degrees	degrees
50	34.9	220	222
60	26.4	210	210
70	17.7	200	199
80	8.9	190	189
90	0	180	180
100	-8.9	170	171
110	-17.7	160	161
120	-26.4	150	150
130	-34.9	140	138

Symons allocates the 19 known coffin lid tables to one of two classes K(7) and T(12), in which the Sothis constellation, with Sirius, is placed 36th and 29th respectively. The five epagomenal days follow the 36th decan, so were respectively either 10/15 or 80/85 days after Sirius.[85] In what follows we will examine the epagomenal stars in the K class.[86]

The possible concept behind the scheme is that for 360 days there was a selection of 36 stars which progressed through 360° in R.A. but only 355° of longitude.[87] In the next five days longitude would reach 360°, but R.A. would change very little.

Table 15 Possible Epagomenal Stars (a/e) with First and Last Decan Stars for –2250 (Last Column – azimuth differences with δ Cma on meridian and 5 days later)

Star	Number K class	Mag.	R.A.	Decl.	Long.	Lat.	Day	Azimuth from meridian	
			degrees	degrees	degrees	degrees		degrees	
α Cma	36	-1.44	55	-20	46	-39	0		
Calculated									
Ideal	a		65	-26	56	-47	10		
Ideal	1		65	-8	61	-29	15		
Possible Stars								meridian	+5 days
δ Cma	a	1.83	65	-28	55	-49	10	0	5.1
TYC6537	b	4.83	65	-25	57	-46	11	-0.5	5.0
TYC5974	c	4.94	64	-20	57	-41	12	0.3	6.3
FW CMa	d	4.14	64	-17	57	-38	13	1.2	7.6
KQ Pup	e	4.82	65	-15	60	-36	14	-0.7	6.1
α Mon	1	3.94	65	-10	61	-31	15	-0.3	7.3

At the same time of day, the five day change in azimuth is between 5° (δ Cma) and 8° (α Mon), while their R.A. is sensibly the same. By having five epagomenal stars instead of a single half-decan, the adjustment is spread over five days, which suggests daily time-

[85] Locher has identified the sceptre of Sothis on a coffin lid as representing a line of stars from β Col to η Lep, which implies a year beginning, not ending, with Sirius – see Von Bomhard A-S, The Egyptian Calendar, Periplus, London 1999, p. 23, Fig. 17. Possibly the image represents another tradition.
[86] Between Crux and Corvus there are many stars where those for the epagomenal days in the J class might be found.
[87] If R.A. and longitude had the same value, the stars would lie on a circle mid-way between the ecliptic and the equator with their declinations and latitudes having the same absolute value but with the opposite sign.

keeping was of paramount importance, but they could tolerate a daily adjustment of little more than 1°. Could they have tried to accomplish this by using offset meridian lines, for the five epagomenal stars? The daily offset would have been successive one-fifths of the overall adjustment. In practice this is not straightforward with these actual stars.

It is easy to calculate the R.A. of each of the 36 decan stars, but without being able to pinpoint their declinations, although –30° would be attractive.[88]As they were evidently prepared to use relatively faint stars, it is not difficult to suggest one for each of the 36 decans. Although, even with a sizeable population to choose from there must have been the odd one which did not fit the scheme precisely. For example with three adjacent stars with 9° and 11° (R.A.) between them, the outer two could be timed on the meridian, but the middle one would be 1° out. To overcome this, they might well have used a pseudo–meridian, one degree offset from the true meridian, for just that one star. Subsequently this could have developed into a grid to cover the area around the meridian, such as can be seen in the Ramesside Star clock of ca. -1470.[89]

At first sight such observations were made by one of two observers, seated facing each other, with the horizontal positions of stars indicated by parts of the other observer's body, such as his eye, ear or shoulder. Neugebauer describes the method as 'incredibly crude'.[90] Perhaps the second observer was only to be imagined, rather as we visualise a clock when indicating directions by the position of an imaginary hour hand. When my oculist says look at my ear, he wants me to look in the direction of his ear, not study it!

From at least the Old Kingdom, Egyptian artists used square grids to set out human figures.[91] It would not be a big step to use parts of the human body to indicate a particular gridline with eye, ear and heart representing the three successive lines from the centre. What angles might have been represented? The proportional distances are in the ratio of about 1, 3 and 6, so if the first line was at one degree, the others would have been about 3° and 6° from the centre.

5. Pythagorean Triangles and ratios of angles, including time, to linear units.

In the Old Babylonian period (ca. 1800 BC), they were well versed in Pythagorean triangles. The Ark tablet contains a value, 14430, for the necessary rope and this can be expressed as 2 x 3 x 5 x 13 x 37, where the last three factors equal the hypotenuse of a Pythagorean triangle.[92] A figure of 2405 (5 x 13 x 37) contains the hypotenuse of no less than 13 Pythagorean triangles – 5, 13, 37, 65(2), 185(2), 481(2) & 2405(4). A circle with such a radius has 108 points with integer co-ordinates, including the four cardinal points.

The more famous tablet, Plimpton 322, has 15 extant rows, each referring to a Pythagorean triangle, although some have argued that the scribe intended to complete a total of 38 rows, covering the edge and both sides of the tablet.[93] There may be good reasons why he stopped at the 15th row.

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[88] Multply class K row number by 10 and subtract 305 to get R.A. in –2250. A star with a declination of around –30°, near the meridian, would move 10° in azimuth over 10° time.
[89] Clagett op. cit. Vol II p.406.
[90] Neugebauer O., op…cit p.561.
[91] Robins G, Proportion and Style in Ancient Egyptian Art, Thames & Hudson, London, 1994, p.59
[92] Finkel I. The Ark before Noah, Hodder & Stoughton, 2014, p 108. No units are actually mentioned.
[93] Brittan J.P. et al, Plimpton 322: a review and a different perspective, Arch. Hist. Exact Sci. (2011) 65 pp 519/566.

The tablet is broken and the rows are incomplete, but it is believed they would have included, in two missing columns, the short side (β) and hypotenuse (δ) of a normalised right triangle with a long side of 1. The first extant column (δ^2) is followed by expanded values b and d and finally the row number.

The 'shape of the triangles varies rather regularly ….'[94] This regularity can be improved significantly.

It is suggested that the operative part was the normalised triangle, with the expanded integer values only required to calibrate an instrument, consisting of an upright of length 1 and a horizontal bar of the same length. The horizontal bar could be moved length-wise, so that the vertical would divide it into two portions with lengths β and 1-β. There would then be two right-angled triangles, sharing a common long side of 1, with sides β, 1, δ, as defined in the tablet, and 1-β, 1, $\sqrt{(2-2\beta+\beta^2)}$ or $\sqrt{(1-2\beta+\delta^2)}$, in the ancillary triangle, which could both be scaled, as required.

Scaling makes no difference to the angles in the two triangles. In the defined triangles the angles change by ca. 0.94° per row, but in the ancillary triangle it is about 1.5°, an attractive 1/60th of a quadrant.

Figures 10 and 11 plot the relationships between the angles and the short sides or the diagonals of the two triangles, several of which are closely linear up to about row 15. The ratios depend on the scaling of the triangles, which is assumed to be by a factor of 11, which is appropriate for the latitude of Babylon (32.5°). There the tangent of the celestial equator (57.5°) is 11/7. The smaller angles in the defined triangles for rows 14 and 15 are 33.3° and 31.9°, with the latter being most appropriate for latitude 31.9°. It has been argued that the tablet was from Larsa on latitude 31.2°, a little south of Babylon.

The ratios of degrees per unit of length are very close to 5° for:

The short sides of both triangles and the smaller angles in the ancillary triangles
The diagonals and the interior angles of the defined triangles.

The diagonals of the defined triangles and the angles of the ancillary triangles have a ratio of about 8°

It would be simple to change the two ratios from 5° and 8° by increasing the length of the long side from 11 to 22 or 44 respectively to give 2.5° and 2° per unit, the two ancient norms. The alternative is simply to reduce the size of the unit of measurement.

If the small angle in the ancillary triangle corresponds to the zenith distance of a star that transits overhead, the ratio of the east/west co-ordinate of the observer's eye is 6° (time to transit) per unit (see last three columns in Table 16 and figure 12). Such stars were known as *zigpu* stars at the time of mul-Apin, ca. 1000 BC.[95]

Plimpton 322 looks like a multipurpose tool for astronomers.

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[94] Neugebauer)., The Exact Sciences in Antiquity, Dover, New York, 1969, p.38.

[95] Hunger H. & Pingree D., MUL.APIN, An Astronomical Compendium in Cuneiform, Archiv fur Orientforschung, Beiheft 24, Horn, Austria, 1989 pp 141-144. Walker C. (editor), op.cit. 1996, p.48 refers to 'A number of Late Assyrian observations and of Late Babylonian eclipse reports are timed in relation to the meridian passage of one of a group of stars known as *zigpu* stars.

Table 16 Plimpton 322- values for rows 1 to 15, after scaling the common long side to 11 units.

	1. Defined Triangle			2. Ancillary Triangle			Stars with Declination 32.5° On latitude 32.5°		
Row	β	δ	smaller angle	11-β	diagonal	smaller angle zenith distance	Time to transit	Position Observer's eye	
	units	units	degrees	units	units	degrees	degrees	units E/W	units N/S
1	10.91	15.49	44.76	0.09	11.00	0.48	0.57	-0.09	0.00
2	10.72	15.36	44.25	0.28	11.00	1.48	1.75	-0.28	0.00
3	10.54	15.24	43.79	0.46	11.01	2.37	2.81	-0.46	-0.01
4	10.36	15.11	43.27	0.64	11.02	3.35	3.97	-0.64	-0.01
5	9.93	14.82	42.08	1.07	11.05	5.55	6.58	-1.07	-0.03
6	9.75	14.70	41.54	1.25	11.07	6.50	7.71	-1.25	-0.05
7	9.33	14.43	40.32	1.67	11.13	8.61	10.21	-1.66	-0.08
8	9.16	14.31	39.77	1.84	11.15	9.52	11.29	-1.84	-0.10
9	8.82	14.10	38.72	2.18	11.21	11.22	13.31	-2.18	-0.14
10	8.42	13.85	37.44	2.58	11.30	13.19	15.65	-2.57	-0.19
11	8.25	13.75	36.87	2.75	11.34	14.04	16.66	-2.74	-0.22
12	7.70	13.42	34.98	3.30	11.49	16.72	19.85	-3.29	-0.31
13	7.38	13.25	33.86	3.62	11.58	18.22	21.64	-3.60	-0.37
14	7.22	13.16	33.26	3.78	11.63	18.99	22.56	-3.76	-0.40
15	6.84	12.96	31.89	4.16	11.76	20.70	24.60	-4.13	-0.48
Overall range	4.07	2.53	12.87	4.07	0.76	20.22	24.03	4.04	0.48
Ratio °/β			3.16			**4.97**			
Ratio °/δ			**5.09**			26.61			
Ratio Ancillary Angle °/ β			**7.99**			16.93			
Ratio angle °/row			0.92			**1.48**			
Ratio Altitude Per E/W unit °/unit							**5.95**		

6. Shadow Lengths - Egypt and Mesopotamia.

Egypt

There are simple portable L-shaped sundials from Egypt dating to the middle of the second millennium B.C.[96] They consist of a short, flat-topped, upright and a long flat horizontal bar to receive the shadow. The gnomon in surviving examples is very short, but some have vertical holes indicating that the height could be raised by the addition of another block. A late hieroglyph even indicates that one gnomon was like a short ladder

[96] Symons S, Ancient Egyptian Astronomy, PhD Thesis, University of Leicester, 1999, pp 127/151. On pp 127/9 she examines one (E1) from the reign of Tuthmosis III, where the distances between adjacent individual hour marks are 1 – 2 – 3 – 4 – 5 with the marks 1,3,6,10, & 15 units from the gnomon.

with 3 different levels.[97] We know that the marks on the horizontal bar are placed, in an arithmetical sequence, at 1,3,6,10 and 15 units from the gnomon. The next two values, in this sequence, would 21 and 28 units, with the latter equally the number of digits in the Royal cubit.[98]

Symons has convincingly argued that those sundials, fitted with a plumb-line to ensure the long bar was horizontal, were designed to be handheld and rotated to point towards the sun.[99] Certainly they could be used in this way, but perhaps more for measuring altitudes rather than estimating time. Comparing the distances, plus or minus 0.5 unit, in the arithmetical series with gnomon height, a gnomon of about 5.5 units would permit good altitude estimates for: 10°, 15°, 20°, 30°, 45°, 60° and 75° (Figure 13).

To measure the same degree values, not of altitude but of time from the rising of the sun, we can calculate the corresponding altitude of the sun at the equinoxes as being: 9.0, 13.5, 17.9, 26.7, 39.5, 51.1 and 60.2. A gnomon of about 5 would give reasonable estimates of time after rising for the first four hours or so. For the remaining hours a shorter gnomon would be required.

We know from the pyramid complex of Pepi II (Figure 9a) that they were particularly focused on the equator, or a little below it. The equinoctial shadow is aligned with the northern edge of the building around the open court. Its eastern edge is about 260 cubits from the centre of the pyramid, equating to 2.6 times the height at ground level. On the roof, if 13 cubits high, the ratio would be 3.0, corresponding exactly to the second mark in the arithmetical scheme. Consequently we can think of the horizontal bar of the sundial as being like that roof, only relatively much longer. [100]

No plumb line is shown in the Osireion drawing and it is suggested that for the estimation of time, throughout the year, the dial was placed due east/west with the face of the horizontal bar flush with the ground.[101] The marks on it could then be extrapolated by eye to the solstice positions (table 17).

Table 17 Latitude 26°, Obliquity of Ecliptic 23.83°, 5 unit gnomon, no allowance for refraction, horizontal bar fixed due east/west and flush with the ground..

	Calculated E/W distance	Arithmetical Scheme							
Hour			Difference	Equinoxes	Summer Solstice		Winter Solstice		
	Units	Units	Units	Hours from rising	Hours from rising	Seasonal	Hours from rising	Seasonal
1	20.8	15	-5.8	1.36	**1.41**	1.24	**1.30**	1.51
2	9.6	10	+0.4	1.94	**2.05**	1.80	**1.83**	2.13
3	5.6	6	+0.4	2.86	3.08	**2.71**	2.63	**3.05**
4	3.2	3	-0.2	4.11	4.55	**4.00**	3.67	**4.26**
5	1.5	1	-0.5	5.32	6.00	**5.28**	4.64	**5.38**

[97] Symons S., op.cit. Figure 19c.

[98] With a one digit gnomon, the altitudes of the shadows corresponding to the first seven positions in the series, would be 45°, 18°, 9°, 6°, 4°, 2.7° & 2.0°. The last, corresponding to one Egyptian Royal cubit of 28 digits, matches one of the two ancient norms in Mesopotamia, with 1 cubit representing 2°.

[99] Symons S., op.cit. p. 143.

[100] The pyramid at Meidum, from ca. 2600 BC has a small chapel on the east and a long causeway, running due east, albeit not horizontally.

[101] Symons S., op.cit. Figure 17, p.131.

The east/west components of the shadows of a 5 unit gnomon, on a latitude of 26°, would be within 0.5 units of four, out of the first five, positions in the arithmetical scheme at hourly intervals (Table 17). The prime reason for the single discrepancy can be attributed to the arithmetical scheme itself, which could easily have had one more position at 21 units from the gnomon, near the end of the bar, for the first hour. The mark at 15 units would indicate 1⅓ hours, not 1, from rising.

The data is broadly consistent with a gnomon of five units on a latitude close to 26° (Figure 13).[102] The dial was evidently intended to indicate seasonal hours, but at the solstices for the first two hours, the times are closer in equinoctial hours. The dial would not show either equinoctial or seasonal hours consistently throughout the year, but was presumably good enough for everyday use.

Once they had recognised that the sun's rays rotated about the top of a gnomon, they could have studied it graphically, just as we can today, albeit with greater ease and precision now. This would explain why refraction seems to have played little or no role. We have already seen above that they were measuring time in units of either 10° or 15° in the pyramid era.

Mesopotamia

Much has been written about the Shadow Length Table in Mul-Apin, but there is one aspect which has still not been resolved.[103] For the equinoxes, no shadow lengths greater than 3 are included, indicating there was an alternative method, other than simply the shadow length, to determine those positions. It was suggested above that in Egypt they extrapolated from the equinoctial positions to those for the solstices. In Mesopotamia they may well have interpolated from the solstices to the equinoxes, graphically by the intersections of the equinoctial shadow path with the straight lines between the points for the two solstices (Table 18 & Figure 15).

Furthest from the gnomon these straight lines mark equal time from rising and lie almost due north/south. Nearer to the gnomon the difference in time from rising, for the two solstices, diverges and the lines deviate further from due north/south. For the first hour or so the table would give quite good estimates of the equinoctial time after rising, but less good thereafter.

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[102] The 26th parallel has interesting properties. Firstly the equinoctial shadow at an altitude of 26.7° is 60° from transit. Secondly, on 26.56°, the equinoctial shadow is exactly half the height of the gnomon from due west/east and the seked (inverse tangent) of the pole is precisely 2. Thirdly, on a latitude of 26.95° and an obliquity of 23.83°, the sun would rise 26.95° either side of due east at the solstices. At a radius of 10 from a gnomon of unit height the north/south distance between the shadows at the solstices would be 17.9 units. With a conventional 180 days between the solstices, each unit would correspond to a decan of 10 days, on average. It is therefore not too surprising that several coffin lid star tables came from Asyut, on latitude 27.2°. (see Symons S., A Star's Year in Calendars and Years (edited by Steele J.M.), Oxbow Oxford 2007), p 2,Table 1.

[103] Hunger H. & Pingree D., MUL.APIN, An Astronomical Compendium in Cuneiform, Berger, Horn, Austria, 1989, pp 153/4.

Table 18. Mul-Apin Shadow Length Table, Latitude 32.5°, Obliquity 23.83°, no allowance for refraction. Indicated times (without brackets) as given in tablet.

Shadow Length	Equinoxes				Summer Solstice			Winter Solstice		
	Shadow length	Time Ind.	Time Calc.	Diff	Time Ind.	Time Calc.	Diff.	Time Ind.	Time Calc.	Diff.
cubits	cubits	degrees	degrees	degrees	degrees	degrees	degrees	degrees	degrees	degrees
10	(8.8)	(7.5)	7.7	**(-0.2)**	6.0	7.6	**-1.6**	9.0	7.9	**+1.1**
9	(7.9)	(9.5)	8.6	**(+0.7)**	6.7	8.4	**-1.7**	10.0	8.8	**+1.2**
8	(7.0)	(10.7)	9.7	**(+1.0)**	7.5	9.4	**-1.9**	11.2	9.9	**+1.3**
7[104]	(6.1)	(12.3)	11.0	**(+1.3)**	8.6	10.7	**-2.1**	12.9	11.3	**+1.6**
6	(5.2)	(14.4)	12.9	**(+1.5)**	10.0	12.4	**-2.4**	15.0	13.3	**+1.7**
5	(4.3)	(17.4	15.4	**(+2.0)**	12.0	14.8	**-2.8**	18.0	16.0	**+2.0**
4	(3.5)	(21.4)	19.0	**(+2.4)**	15.0	18.2	**-3.2**	22.5	19.8	**+2.7**
3	3	25.0	22.0	**+3.0**	20.0	23.7	**-3.7**	30.0	27.4	**+2.6**
2	2	37.5	32.0	**+5.5**	30.0	33.7	**-3.7**	45.0	43.1	**+1.9**
1	1	75	57.0	**+18.0**	60	55.8	**+4.2**	90	73.7	**+16.3**

There is no doubt that the mul-Apin table referred to equinoctial time after sunrise, but there remains the problem with the one cubit length for the winter solstice. For the summer solstice and the equinoxes the length of shadow, when respectively 60° and 75° from rising, would be 0.9 and 0.7 cubits, both close enough to be rounded to 1 cubit. At the winter solstice the shortest shadow is 1.57 cubits, on the meridian, but it is only 74° from rising and therefore far from the 90° of the constant. It is reasonable to consider that it 'was presumably added for reasons of symmetry and to show the value of the constant for that solstice' or the measurements were a little further south.[105]

Hunger and Pingree claimed that 'we must regard the table as based on mathematical manipulation rather than on observation'.[106] Clearly the table incorporates reciprocal relationships, but they must also have had a deep practical understanding of the underlying phenomena (Table 18). The values in the table are after they were forced into the straight jacket of the formulae and so it is likely their underlying data was much more precise. For the equinoxes the fit is not close, presumably because of the 'desire to fix the constant (75) midway between those for the solstices (60 and 90)'.[107]

From Table 18, for a shadow length of 2 cubits at the solstices, the product of the shadow length and the calculated time after sunrise is 67 and 86, compared with the scheme constants of 60 (summer solstice) and 90 (winter solstice).. Figure 16 shows the linear relationship between time and the inverse shadow length and the solstices and equinoxes. For the solstices the linear trendlines indicate ratios of 96 and 56 and also rising H.A. of 256° and 286°, which correspond to declinations of 20.8° and -23.4° and rising azimuths of 65° and 118°. The good fit of the latter, ignoring the 1 cubit value, suggests that the scheme was based primarily on the winter solstice with a constant of 90 and that the 60 and 75 for the summer solstice and equinoxes were derived therefrom.

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104 The table shows no values for this shadow length, because of the difficulty of dividing by 7 in the sexagesimal system, but it is included here for completeness.

105 Bremner, R.W., The Shadow Length Table in Mul.Apin, in Die Rolle der Astronomie, Graz, 1993, p.370. See also Steele J., Shadow-Length Schemes in Babylonian Astronomy, Academia, 2012?, p.11: 'This entry in the scheme is therefore an artefact of the underlying mathematical rule and is, presumably, included in the text either simply for the sake of completeness or perhaps because it is the value of the constant c for that month and so is useful in calculation.' A little south of Babylon the shadow would be under 1.5 cubits, which could be rounded to 1.

106 Hunger H & Pingree D, Astral Sciences in Mesopotamia, Brill, 1999 p.80.

107 Bremner R.W., op.cit. p.369.

7. The 2:1 and 3:2 ratios for longest to shortest day and the Path of Anu.

People all over the world have used the rising and setting of the sun as markers for annual events such as the standstill positions at the solstices.[108] Those living in what is now northern Iraq were surely no different and would have noted the extreme positions of the sun at the horizon. They would soon have realised that these four points, plus the meridian, divided a circle into six equal segments. Adding in the east/west line of the equinoxes gives segments of 30° and we have noted such bearings at Eridu (Latitude 30.5°) around -5000 (Page 12 above). By c.-3100 they were using a star pictogram with 8 points, so by then they were thinking in segments of 15°.

Figure 17 shows graphically the 2:1 and 3:2 ratios for the longest to shortest days, based respectively on azimuth and equinoctial time, at the horizon. The outer time polygon has sides of 24 cubits for 36° time. Interestingly the angle, between the solstices and the equinox, is 18°, similar to that of the oblique palace wall (17°) and to the divisions between the paths of Anu, Enlil and Ea (see footnote 15 above).

Both estimates (15° declination and 17° from due east) for the boundary of Anu stars would be correct on a latitude of 28°, which suggests that the width of the Anu band was more likely to have been determined in the southern, rather than the northern, part of Mesopotamia. Table 19 shows the situation on a latitude of 30° and demonstrates that Anu's limits were probably based on equinoctial times above the horizon with the width being 36° or one tenth of a day.[109] Figure 16 shows the limits of 7 units from the east/west line for the Anu band on a latitude of 35°

Table 19 Latitude 30°. Obliquity of the Ecliptic 23.9°. No allowance for refraction.

Declination	Rising HA	Time above horizon	Rising Az	Azimuth swept
degrees	degrees	degrees	degrees	degrees
23.9	255	210	62	236
Anu 15	261	198	73	214
0	270	180	90	180
Anu -15	279	162	107	146
-23.9	285	150	118	124
Anu range	18	36	34	68
Solstice range	30	60	56	112
Solstice ratio[110]		1.4		1.9

Each 24 cubit side corresponds to 36°(time), giving a ratio of 1.5° per cubit, which with a double cubit would increase to 3.0°. Such a unit would approximate to the ratios implicit

[108] Thurston H., Early Astronomy, Springer-Verlag, New York, 1994, pp 10/11.
[109] On a latitude of 35°,lines of stars with declinations of ± 15° would rise 18° from due east and their time above the horizon would be 202° and 158°. The tangent of 18° is 1/3, which would have been an attraction.
[110] The ratios of 3:2 and 2:1 on latitude 35° were 2.8:2 and 1.9:1 on latitude 30°.

in HS345, summarised as 51 units from the 'Stars' to Bootes and a further 7 units to Scorpio.[111] This is particularly true if the 'Stars', in this instance, should be identified not as the Pleiades but as the Hyades, at least for the overall distances to SUPA and the Scorpion.

Table 20 Summary of tablet HS245, the Hilprecht Text (R.A. for -1600)

	Exemplary Star	R.A.	Degrees from Hyades	Distance	Ratio
		degrees	degrees	units	degrees/ unit
Stars	η Taurus (Pleiades)	8	-11		
	θ Taurus (Hyades)	19	0		
SUPA	α Bootes	172	153	51	3.0
Scorpion	α Scorpio	197	178	58	3.1

Table 21 summarises the evolution of ideas about the ratio of the longest/shortest day. It does not include HS245, which pushes the 3:2 ratio back to the Old Babylonian period..

Table 21 , Horizon measurements on Latitude 35°, Obliquity 23.9°, no allowance for refraction

	Azimuth swept	Cubits swept hexagon	Cubits swept Stepped curve	Hourlines Hor. Dial	Text BM17175 + 17284[112]	Text mul.Apin	Text mul.Apin	Text Ivory Prism	Length of Daylight
	Degrees azimuth	Cubits along sides	N/S cubits	Degrees from meridian	none	minas	beru	beru	Degrees time
Approx. date	-5000?	?	<-700		-1800	-1000	-1000	<-610	
S. Solstice	240	96	96	120.7	4	4	3.6	8	216
Equinox	180	72	72	90	3	3	3	6	180
W. Solstice	120	48	48	60.4	2	2	2.4	4	144
Ratio	2:1	2:1	2:1	2:1	2:1	2:1	3:2	2:1	3:2
Ratio degrees per unit	1	2.5	2.5	1	60	60	30	30	36

In 1947 Neugebauer was clearly taken with the idea of the 2:1 ratio for the longest and shortest days being based on the use of a water clock, but by 1975 he was rather more cautious.[113] He refers to 'the assumption that the given weights represent the outflow of water from the bottom of a cylindrical container...'. It was only an assumption and in 1996

[111]Hunger H. & Pingree D., Astral Sciences in Mesopotamia, Brill, Leiden, 1999, p.54
[112]Hunger H. & Pingree D. Mul.A[pin, AfO, Horn, Austria, 1989, p 163.
[113]Neugebauer O., The Water Clock in Babylonian Astronomy, 1947, ISIS 37, pp37/43 and HAMA, 1975, p 708.

Hoyrup drew attention to the problems with the water clock model.[114]In 2000 Michel-Nozieres concluded that 'the water weight data … cannot be taken literally'.[115] In spite of Hoyrup's work, Hunger and Pingree in 1999 stated that '1 mina of water in a water-clock measured a third of an equinoctial night', with no caveats.[116]

In mul-Apin the ratio is associated with minas, normally a measure of weight, equivalent to about 500 gms.[117] From school problems from about -1800 we learn of water flowing from a water-clock. However the existence of water-clocks does not mean that a ratio established over millennia, was immediately discarded.

The study by Michel-Nozieres of the problems inherent in outflow clocks found that under the best conditions, the ratio would approximate to$\sqrt{2}$:1, which is far from 2:1. In fact , expressed as 2.8:2, it is obviously much closer to the 3:2 ratio in time.

The 2:1 ratio appears later (pre -611) on an ivory prism as a ratio of angles, expressed in beru (30°) and us (1°), so this same ratio was, in different texts over more than a millennium, based on unstated units, units of weight and units of angle or time. We also have to bear in mind the use of ninda, normally a unit of length, in mul.Apin. After the summer solstice (II I 11/12) 'the sun … turns and keeps moving towards the South at a rate of 40 NINDA per day' and after the winter solstice (II I 17/18) 'the sun … turns and keeps coming up towards the North at a rate of 40 NINDA per day'.[118] In the same section there is reference to the length of the watch in terms of minas, so we appear to have a mixture of units of weight and length.

If, at the time of mul=Apin and before, they could measure time accurately enough in equinoctial units to confirm the 3:2 ratio, it seems somewhat perverse to use simultaneously a 2:1 ratio of weights, unless the two ratios were never intended to refer to the same phenomenon or were not established at the same latitude.

To resolve this issue perhaps we need to take a different approach. When experimenting with water clocks they might have tried weighing the water dripping into a bowl until the scales tipped.[119] This would justify measuring the quantity of water by weight rather than volume. If they were measuring the time for the sun to traverse a large segment of the horizon they might have noticed that it was like the bow wave of a swimming duck. This would justify the association of weight and ducks, with many standard weights being in the form of a duck.[120] However it would imply that 'mina' in addition to its usual meaning of weight was also a segment of a circle. With 6 minas in a full day, each would correspond to 60°.

[114]Hoyrup J., A note on water-clocks and on the authority of texts (pre-print 1996), AfO, 44-45,
[115]Michel-Nozieres C., Second Millennium Babylonian Water Clocks: a Physical study, Centaurus 2000, Vol.42 pp 180/200.
[116]Hunger H. & Pingree D., Astral Sciences in Mesopotamia, Brill, Leiden, 1999, p.46
[117]Hunger H. & Pingree D, Mul.Apin, An Astronomical Compendium in Cuneiform, AfO, Horn, Austria 1989, pp 163/4 (Appendix). The tablets are dated to the old Babylonian period c.-1800.
[118]Hunger H. & Pingree D., Mul.Apin op.cit pp 72/75.
[119]In Portugal many years ago I saw an old domestic water meter which used such a system. When one bowl filled the flow was diverted to fill the other. Each switch being counted to determine the volume.
[120]A water clock with sinking bowls would also remind them of ducks, with both likely to dive suddenly .

8. Djed Pillar and Time Measurement.

The vertical Djed pillar in Figure 14 vaguely hints that it might be related to the measurement of time using a horizontal sundial. On the other hand Figure 18 shows a modern drawing of the hour-lines for an east-facing vertical sundial with a style aligned to the pole and also a Djed pillar at Abydos (latitude c. 26°) inclined at c. 25° from the vertical and surmounted by twin plumes.[121] The two are remarkably similar. The width of the 'pillar' corresponds to the length of the style and the hour-line positions depend on the height of the style away from the meridian plane. In this type of vertical dial the longest shadows are at mid-day and the shortest at the horizon. The 'pillar', on which the shadows fall, is inclined from vertical at an angle corresponding to the latitude of the site.

The Djed pillar symbol itself dates back to pre-historic times, but this does not imply that it was always associated with the measurement of time.[122] It could be that when this type of sundial was developed, someone noticed that the shadow lines looked like a leaning Djed pillar, whatever that might have been. The ritual of 'raising the djed pillar', is known from the Old Kingdom at Memphis, which suggests the possibility that the association with time was established by say 2500 BC.[123] This date coincides with the growing importance of the east/west line (cf Menkaure's pyramid causeway) and the size of the mortuary chapels and other buildings immediately east of pyramids.

The Djed pillar symbol, and presumably its dialling properties, reached Mesopotamia from Egypt around -1800[124]. From about -500 there is a shadow table (BM29371) with intervals of 5 days, against each of which is written 'One cubit shadow, 1 $^2/_3$ double-hours day'.[125] This has been interpreted as meaning 'after 1 $^2/_3$ double-hours of day the shadow of the gnomon has a length of 1 cubit', throughout the year. If 1 $^2/_3$ double hours equates to 50° (time), then an east-facing vertical gnomon with a style of $^5/_6$ cubit, would have a shadow of 1 cubit.[126]

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[121]Cousins F.W., Sundials, Redwood Press, Trowbridge, 1972, p.132 and Lurker M., The Gods and Symbols of Ancient Egypt, Thames and Hudson, London 1982, p.47. There is a large ancient Greek sundial with similar curves at the British Museum (ref:1816,0610.186). It is inscribed 'Phaidros, son of Zoilos'.

[122]Shaw I. and Nicholson P., British Museum Dictionary of Ancient Egypt, London, 1997, p.86. On page 304, they mention the possibility that the was sceptre was used as a gnomon and it might be seen as stripped down version of a vertical dial, facing east or west, with the angled head pointing to the pole.

[123]Lurker M., op.cit, p 47.

[124]Black J. & Green A., Gods, Demons and Symbols of Ancient Mesopotamia, British Museum, 1992, p.74.

[125]Britton J. & Walker C., Astronomy and Astrology in Mesopotamia (in Astronomy before the Telescope), British Museum, 1996, p.47. More recently in Steele, J. Shadow-Length Schemes in Babylonian Astronomy, Academia, 2012? pp 30ff there is a different interpretation of the text.

[126]The calculation is Tan 50 x 5/6.

Summary Timeline

Egypt in red

Year	Location	Pythagorean Triangles	Subdivisions of circle	Time
-5500	Tell es-Sawwan		45°	
-5100	Eridu		30°	
-4700	Nabta Playa	3,4,5		
-4500	Egypt			Year 360 +5 days
-4450	Nabta Playa		26.56°	
-4250	Eridu	3,4,5		
-3900	Abydos		5 pointed star	72° divisions
-3100	Mesopotamia		8 pointed star	
-2600	Saqqara	4 different		
-2556	Khafre's pyramid Giza			Built-in hours 60 minutes
-2500	Menkaure's Causeway?			Djed sundial
-2500	Standard Pyramids	3,4,5		Standard hour 60 minutes
-2300	Coffin Lids Many ex Asyut			Short hour 40 minute s
-1900	Mesopotamia Old Babylonian Period	26 different - Ark & Plimpton tablets		Djed sundial daylight 2:1 & 3:2 ratios
-1500	Egypt			L-shaped sundials
-1000	Mesopotamia Mul-Apin			Shadow length table
-700	Babylon		azimuth in 2.5° steps	Longitude near horizon

Index