Babylon: Linear Measures of Celestial Angles and an Observatory

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new section on Rogem Hiri p.61

Abstract

The paper examines Babylonian records, from the 1st millennium B.C., of planets passing fixed stars and specifically their up/down differences in linear cubits. It shows they were using the top of a gnomon as a foresight around which the observer moved on non-circular arcs, where the ratio of degrees per cubit was 2.5° (azimuth). Particularly near the horizon they were able to ensure close alignment in longitude between the star and the planet. The up/down measurements were then almost identical to the distance between the two bodies, using a straight rule. Finally an area north of the Western Court of the Southern Palace is identified as a possible site of the observatory. Appendix A gives a worked example and Appendix B looks at other earlier developments in the region. It is followed by a Timeline and Index.

This study is based on the surviving Babylonian records of planets passing fixed stars in the years from –418 to –73 BC .[1] The collection includes 1049 passages, where the up/down differences were recorded in linear units with a maximum of 6 cubits and a minimum of 1 finger (1/24th cubit). There were also 27 records of similar before and after differences. The recorded information was laconic with an up/down report for the year –418 reading simply 'Month II, night of the 9th, Mars was 4 cubits below θ Leonis'.[2]

Not all the possible combinations of fingers and cubits are represented. There were certain preferred values with rounding errors running from +/- ½ finger, for distances under 6 fingers, to +/- 6 fingers for those above 4 cubits. In percentage terms such errors reach +/- 50% with a minimum of about +/- 3%. For distances below 1 cubit, the percentage is between 7 % and 50% and above 1 cubit between 3% and 8%.

There are gaps in the longitudinal coverage of the Normal Stars and, consequently, in declination. In terms of azimuth, the gaps, near the horizon, can be seen in Figure 2.

Professor Jones concluded that the up/down cubit values were related to differences in latitude and found the mean ratio of degrees (latitude) per cubit to be about 2.3°, which lies between the two ancient norms of 2° and 2.5°.

Ptolemy, in his criticism of the data, provided clues about how the measurements were made:

In general, observations [of planets] with respect to one of the fixed stars, when taken over a comparatively great distance, involve difficult computations and an element of guesswork in the quantity measured, unless one carries them out in a manner which is thoroughly competent and knowledgeable. This is not only because the lines joining the observed stars do not always form right angles with the ecliptic, but may form an angle of any size (hence one may expect considerable error in determining the positions in latitude and longitude, due to the varying inclination [to the horizon frame of reference]); but also

[1] Jones, A., A Study of Babylonian Observations of Planets Near Normal Stars, Arch. Hist. Exact. Sci. 58 (2004) pp.475-536. I am very grateful for Professor Jones giving me access to his Collection A and also to my son, Geoffrey, for help with the drawings and his patience.

[2] Sachs A.J. and Hunger H., Astronomical Diaries and Related Texts from Babylonia, Vienna 1988, Vol. I, p.63

because the same interval [between star and planet] appears to the observer as greater near the horizon, and less near mid-heaven;[footnote] hence, obviously, the interval in question can be measured as at one time greater, at another less than it is in reality.

[footnote] This appears to be the only reference to the effect of refraction (if that is what it is) in the Almagest, despite its obvious relevance to the observations of Mercury's greatest elongation….[3]

Clearly he considered they were thinking in terms of the ecliptic, but were also assuming lines of longitude and latitude were always at right angles.

To investigate how and what they were measuring two main assumptions were made:

1. The observer used a linear measuring rod to determine the up/down or before/after position of the planet in relation to one of the 28 so-called Normal stars. He did this by aligning one end with the star and the other point with the planet, using the top of a vertical gnomon as a foresight (Figure 1).[4] The rod may have been hand-held or fixed in a rest. For each passage, the ratio of degrees (latitude) per cubit was converted to DTOG, the distance of the observer's eye from the top of the gnomon.[5] The mean ratio of 2.3° per cubit implied that his eye was about 25 cubits from the top of the gnomon, which equates to ca. 13 metres and gives an idea of the size of the device, assuming a cubit of about 52 cms. Any errors in measurement and recording, including the not inconsiderable rounding errors, are accumulated within the DTOG value.

2. In 96% of the surviving records the difference in longitude (planet less star) was between –3.3°/+3.8°. Consequently the distance between the two bodies would be only marginally greater than their difference in latitude. By repeated iterations 656 passages (63% of the surviving records) were found where the distance between the two bodies was within 0.2% of the recorded up/down distance in cubits.[6] The margin of 0.2% is small but it equates to a 3.6° difference in the before/after positions, if a rectangular co-ordinate system was used, as Ptolemy indicated.[7]

Professor Jones calculated the ecliptic co-ordinates of the outer planets for midnight and those of the inner planets about 4 hours, either before or after midnight.[8] No adjustments were made either to these celestial co-ordinates or for refraction.

Appendix A has a worked example of the calculations for one passage.

[3] Toomer G.J., Ptolemy's Almagest, Duckworth, London, 1984, p.121.

[4] The main justification for such an arrangement is that it brings the measuring scale close to the observer. It is not essential as the observer could be at the centre of the device with the scales about 13m away, but in that case it is hard to accept the 'measurements' as much more than estimates.

[5] DTOG, distance from top of gnomon, equals 1/sine(ratio degrees per cubit)

[6] Varying the altitude of the two bodies changes the linear distance between them. Out of the 128 passages with up/down distances of 4 fingers or less and rounding errors greater than 12.5%, only 29 had the distance apart within 0.2% of the recorded value.

[7] Using plane trigonometry, Cosine (3.6°) = 0.998.

[8] Jones A., op cit. p.481 gives UT 21 for the outer planets and either 17 or 1 UT for the inner planets.

Before and After Alignment

There has been considerable discussion about their ability to measure in ecliptic co-ordinates.[9] Of the 656 passages 59% were most closely aligned in longitude particularly at lower altitudes. In 78% of these passages, the longitude difference (planet less star) was less than 1°, compared with 63% of all surviving records. Other passages were better aligned in R.A. (34%) or even azimuth (7%) (Figure 2). This confirms Professor Jones's conclusion that they were thinking in ecliptic co-ordinates, but, it now appears, their alignments, in longitude, were closer at lower altitudes. The mean altitude of the stars for the passages, best aligned in longitude, was 7.5° and, for the others, 16.6°.

Positions of the Observer's Eye

The passages, best aligned in longitude, were sorted to the order of the star's azimuth, in the west and the east. The relationships between azimuth and, separately, the north/south and east/west cubit co-ordinates of the eye of the observer are shown in figure 3.

Surprisingly the relationship between azimuth and the north/south co-ordinates is very close to linear, with each cubit corresponding to 2.5° of azimuth, which implies that the paths of the observer were neither circular arcs around the gnomon nor straight lines.[10] Instead those paths must have been stepped arcs.[11] This provides a good indication of the intended paths of the observer in the east and west.[12] However, in practice, within 20° of due east/west, the divergence from a straight line is less than 1 cubit and could well have been ignored, if a straight line was more acceptable.

The observer's position in the vertical is generally within 6 cubits of the top of the gnomon, but drops to 10 or 11 cubits in places (Figure 4). There is a notable anomaly about 5 cubits north of the gnomon, where, particularly in the east, the observer's eye drops down to about 10 cubits. This anomaly also marks a sharp fall in the number of passages, when the observer is between 5 and 10 cubits north of the gnomon.

From the foregoing we can deduce that there was a structure around the gnomon which facilitated observations where the observer's eye was within 6 cubits of the top of the gnomon.

There are other aspects brought out by the moving mean lines in figure 4.[13]

[9] Hunger H. & Pingree. D. Astral Sciences in Mesopotamia, Brill, 1999, p.269.

[10] This also confirms that they were thinking in terms of 2.5° per cubit.

[11] I am most grateful to P. Starkey, a neighbour and mathematician, for providing the modern polar equation for such curves: $r \sin \theta = (Y_{max} .2/\pi)\ \theta$, where r is radius and θ the angle in radians. Spiral curves may have been used in Egypt at an early date (see Appendix B). A similar linear relationship, but closer to 2.6° (azimuth) per cubit, was found for those passages best-aligned in R.A., indicating that for those passages the observer was slightly closer to the gnomon. As they were also higher, it implies that the observer's path, in cross-section, was like a steep-sided bowl.

[12] A target ratio of 2.5° per cubit implies the distance to the top of the gnomon on the east/west line was about 23 cubits (1/22.9 = Tan 2.5°). From there it is simple to calculate thirty-six cubit steps, each of 2.5°, to the north and south. With due north/south being at 0,36/0,-36 and due east/west at ca. 23,0/-23,0. The intermediate positions at 45° are +/-18, +/-18. With 36 steps the sum of each successive hypotenuse totals 44.6 cubits, so along that path each cubit averages about 2°. In practice the steps may have been irregular and larger than the 1 cubit assumed. If used to measure altitude, rather than azimuth, such a curve would resemble the recumbent crescent moon, a common motif in Mesopotamia, but an impossible position for the moon in practice.

[13] Moving means help to smooth out erratic data, but depend on how the data was sorted. In figure 4 it was in order of N/S cubits, but in figure 6 in order of azimuth.

A Possible 'Observatory'

To visualise the observatory, we might think of a 6 cubit gnomon standing above the flat roof of a building with the much lower areas corresponding either to the ground outside or to interior open courtyards. In the Southern Palace at Babylon, there are many such courtyards, but there is an area north of the Western Court of particular interest. Before the whole of the area had been excavated, a part to the north-east was described as follows:

The houses of this part of the palace are remarkable for the strength of their walls and the admirable regularity with which they are laid out. Court 38 is reached by a passage-way from the Principal Court, the latter through a hall, as in the case of 25, 26 and 27, opens with three doors on to court 38. Between the doors, pillars project from the walls and correspond with others on the opposite side. They must have served as piers to support arches for the ceiling, although it is difficult to make out clearly what was the object of this structure.

The roof of this area of the palace was evidently intended to support more weight than usual. It may appear improbable that an observatory would be rectangular, but we can perhaps think of it as being like graph paper. Today we use Mercator charts, with rectilinear lines of longitude and latitude, and also Ordnance Survey maps with a rectangular grid. It is a question of balancing the pros and cons of such arrangements.

There are circles in the sky which produce straight lines, aligned with the cardinal directions, on the ground. Firstly there is the meridian. Secondly a prime purpose of an observatory would have been the measurement of time both at night and during the day. In a horizontal sundial the hour-line for 6 hours to transit runs due west/east through the pole. Thirdly the shadow of the sun, at the equinoxes, runs due west/east just north of a gnomon.[14] We thus have three perfectly straight lines – the meridian, the hour-line for six hours to transit and the shadow of the sun at the equinoxes – and we have already noted that the stepped arcs run sensibly due north/south within 20° (8 cubits) of due east/west. Together these lines form a near rectangular outline for observations.

Just south-east of court 48 is a short length of wall of abnormal width (1.8m), which is aligned with a passage leading from the northern wall of the palace. None of the other similar passages, running due south, from the oblique northern wall, is so short.[15] The wide wall and the short passage may perhaps have marked the meridian.

The short thick wall links two substantial east/west walls, about 5m apart; one just south of court 48 and the other north of the transverse corridor.[16]

[14] With a gnomon of 1 cubit, on a latitude of 32.5°, the pole would be 1.57 cubits to the south and the equator 0.637 to the north, with the distance between them being 2.207 cubits.

[15] The oblique northern wall of the palace is stepped, both vertically and horizontally, and is inclined about 17° from east/west. The 17° of azimuth matches that quoted for the limits of the path of Anu in Walker C. (editor), Astronomy before the Telescope, British Museum Press, 1996, p.48. It corresponds to the rising/setting of stars with a declination of +/- 14.3°, which is close to the 15°, for the Path of Anu, quoted in Hunger H. and Pingree D., Astral Sciences in Mesopotamia, Brill, Leiden, 1999, p.61.

[16] The two east/west walls may perhaps be linked to anomalies in Figure 4. Such walls would prevent the observer going lower for higher altitudes, and would oblige him to move nearer the gnomon. The most northerly of the two walls is aligned to the well-head NW of court 47 and may have carried a water conduit.

The stepped curves, with each north/south cubit corresponding to 2.5° of azimuth, would fit within the north/south width of this part of the palace, with the gnomon about midway between the two east/west walls. However, as we will see, there are reasons to believe it was perhaps ca. 2m further north. In figures 5 & 6 it is on the east/west wall just south of the two courts, 39 and 48.

Figure 5 shows:
the paths of the tip of the sun's shadow at the solstices, equinoxes and for those stars that transit overhead,
the hour-lines around the pole,
the stepped arcs,
bearings around the gnomon and
radial distances from the gnomon[17] Radial distances formed part of an older table of shadow lengths.[18]

Celestial and associated phenomena influenced the layout in this area of the palace. Junctions are marked in figure 5 by small circles of radius 0.5 cubits or about 26 cms.

The following table refers to the room immediately north of court 39.

Table 1.

Corners of Room to north of court 39	
Location	
SW	3rd hour-line from transit
Exit to south	Azimuth 45° and Stepped arc at 18(N),18(E) cubits from gnomon
SE	Radius 30 cubits and Winter solstice shadow
NE	Azimuth 45°
Exit to north	Radius 30 cubits
NW	Azimuth 30° and Stepped arc at 24(N) cubits from gnomon

With a 6 cubit gnomon, the line of the equator would lie above the passage linking the two courts and the pole would be on the more southerly of the two parallel walls. The equator coincides with the anomaly noted earlier (Figure 4). Furthermore the transit shadow of the sun at the winter solstice would fall on the end of the short passage running south from the city wall. The NW corners of both courts would be on a bearing of 45° from the gnomon.

The proposed site seems plausible, even though having the gnomon in such a position is fraught with problems, caused by the many towers and turrets, particularly those around the palace itself. They were slender, but high and closely spaced, so that they would appear like a solid wall, if viewed obliquely.[19]

Figure 2 shows that there was an almost complete dearth of passages, near the horizon, between bearings of 6° and 22° from due east/west. To the south-east the large gateway

[17] Berossus is considered to have invented the hemicycle sundial around 300 B.C. (Cousins F. W., Sundials, Redwood Press, Trowbridge, 1972, p. 72.)

[18] Hunger H. & Pingree D., Mul-Apin, Horn, 1989. p 153/4. The shadow length table is discussed in Neugebauer O., A History of Ancient Mathematical Astronomy, Vol. I, Springer-Verlag, Berlin, 1975, p 544/5, by Bremner R.W., Die Rolle der Astronomie in den Kulturen Mesopoatmiens, Symposium, Graz, 1991, pp 367/382 and by Hunger H. & Pingree D., Astral Science in Mesopotamia, Brill, 1999. pp 79/82. See also Appendix B, page 39.

[19] The turrets were closely spaced and with a width of about 6.5m. Viewed from within an angle of about 50° there would be no visible gaps between adjacent turrets.

between the Central and Principal Courts is on a bearing 12/19° from due east and could well have blocked the view to the horizon. To the south-west there is the Western Citadel, where maybe there was a similar high structure.

To check alignments at night, the observer would need to get his eye down to base level. A schematic drawing shows the palace roof as flat, but with the major north/south walls projecting above roof level.[20] In the area of the proposed observatory, the tops of all the walls were, perhaps, raised to 1.5 cubits above roof level with the gnomon 6 cubits higher still.[21] The main level at 6 cubits below the top of the gnomon would receive the shadow of the sun and, at night, the eye of someone sitting on the roof itself would be in the same plane.[22]

An additional platform, 3 cubits below the top of the gnomon, would enable the observer to measure on the horizon. He could further adjust the level of his eye by standing on a block or by kneeling[23] In the two open courts the observer would be able to go much lower.

Even if the observer was meant to stick to the designed paths, there would be nothing to prevent him making observations wherever he could get a sight of both the gnomon and the celestial bodies.

The moving means of the positions (Figures 4 & 6)), show that in both the west and east, the observer's path was generally close to the stepped arcs. On both sides, near the path of the sun at the summer solstice, the positions of the observer are closer to the gnomon, than indicated by the stepped curve (Figure 6). There may, perhaps, have been some sort of track marking the shadow of the sun at that extreme, preventing the observer going deeper for higher altitudes and obliging him instead to move nearer the gnomon. In such cases Ptolemy's remark about the same interval (angle*)* appearing *'to the observer as greater near the horizon, and less near mid-heaven*' would apply.

Passages, where the depth was more than 6 cubits, are shown by heavy lines, notably in the north and due east and west of the gnomon (Figure 6). In the north-west the observer was at a significant depth over what appears to be a large area of solid brickwork, but it could have been modified without leaving a trace in the archaeological record.[24] On the east the depth was also significant in the south-east corner of court 39.

The anomaly, 5 cubits north of the gnomon, can be linked to the two courts and lends credence to the suggestion that those passages, well-aligned in longitude, were recorded around a gnomon in the position indicated. This is difficult to prove though, especially in

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[20] Koldewey R., The Excavations at Babylon, London, Macmillan, 1914 fig. 87. Shows cross-section through walls north of the Southern palace, with the roof of the palace shown schematically. Fig. 43 shows a birds' eye view of Southern Palace, with only some of the main walls rising above roof level.

[21] The gnomon would be 7½ cubits above the roof, which would shift the line of the sun's equinoctial shadow, from the centre of the passage, to the gnomon side of the passage wall.

[22] In the XVIII century the Jai Prakash Yantra at Jaipur similarly had complimentary sections of the two bowls cut away to allow the observer to get his eye into the plane of the bowl. (Rajawat, D.S, Jaipur's Jantar Mantar, Jaipur, date ?, pp 49/53)

[23] Analysis of the depths below the top of the gnomon suggests there was a very slight preference for certain depths: –0.5, -2.5, -3.5, -5, -6, -7, -9 and -11 cubits, but only 18% of passages were below -6 cubits.

[24] Koldewey R & Wetzel F, Die Konigsburgen von Babylon, WVDOG54, Leipzig 1931, Die Gebaude 39 und 48 Nordlich vom Westhof. I am grateful to Helene Lambrinudi and Andreas Kindler for translations from the German.

the face of evidence that observers were employed by the Temple of Esagil, a long way south of the Southern Palace.[25]

Measurements

Finally we must consider what they were actually measuring..

The 656 passages were divided into six groups, according to the alignment of the two bodies in longitude, R.A or Azimuth and then whether they were observed in the east or west. For each passage the angle, in the vertical plane between the star and planet, was calculated. This angle has been termed the alignment angle and figure 7 shows how it varied with longitude. The two dashed curves are calculated values, assuming perfect alignment in longitude and with the lower of the two theoretical bodies at an altitude of 2°.

The rod, shown schematically in figure 8, would serve to check the alignment in longitude and to measure differences in latitude, assumed to be at right-angles.

Conclusions

They were using an observatory, originally laid out for the accurate determination of azimuth in linear cubits (2.5° per cubit) measured along lines parallel to the meridian. The observer would move along non-circular arcs, around the gnomon, and would be at a varying distance from the top of the gnomon. Consequently the ratio of degrees (except azimuth) per cubit would also vary.

In attempting to work in ecliptic co-ordinates, they recognised the difficulties involved. To reduce these to a minimum, they aimed to measure latitude only when they were sure the two bodies were closely aligned in longitude and this was easier close to the horizon. With close alignment in longitude, the distance between the two bodies would represent their difference in latitude.

The layout of the area to the north of the Western Court seems to have been influenced by celestial and related phenomena. It is possible, but not proven, that the measurements could have been made there.

[25] Hunger H. & Pingree D. op.cit p.139.

Appendix A – worked example

Table 2.

1		**Data from Collection A**[26]		
2	Star		Planet	Difference Planet less Star or common value
3	α Virgo	Year –270/10/21	Mars	
4	172.316	Longitude °[27]	172.052	-0.264
5	-1.906	Latitude °	1.098	3.005
6		Up/Down cubits		1.5
7		Degrees Latitude per cubit		2.003
8		**Calculated Values for two bodies** Spherical trigonometry		
9		DTOG cubits - 1/Sine(row7)		28.608
10	172.197	R.A. ° (Latitude 32.5° and Obliquity of ecliptic 23.728°)	173.156	-0.958
11	1.336	Declination °	4.197	2.861
12	1.659	Altitude ° found by iteration	4.000	2.341
13	271.115	Hour-angle ° (transit 360°)	272.074	1.222
14	-0.528	Azimuth from 90°	-2.432	-1.904
15		Sun's Longitude > planet's, so passage in east & observer to west of gnomon		
16		**Calculated Positions – Observer's eye** Plane trigonometry		
17	-28.594	X cubits West (-) East (+) =DTOG x Cos (row12) x Cos (row 14)	-28.512	-0.082
18	-0.264	Y cubits South (-) North (+) =DTOG x Cos (row12) x Sine (row 14)	-1.211	-0.948
19	-0.828	Z cubits below horizontal =Row 20 x Sine(row 12)	-1.991	-1.163
20	28.596	Horizontal radius from gnomon $\sqrt{(X^2+Y^2)}$	28.538	
21		Horizontal distance between two positions – cubits = $\sqrt{(\text{Diff } X^2 + \text{Diff } Y^2)}$		0.951
22		Total cubits between two positions = $\sqrt{(\text{Diff } X^2 + \text{Diff } Y^2 + \text{Diff } Z^2)}$ Compare with recorded 1.5 cubits row 6		1.502
23		Bearing in horizontal plane from North ° = ArcTan(Diff Y/Diff X)		-85.042
24		Absolute alignment angle in vertical plane between two positions ° = ArcTan(Diff Z/row 21)		50.726

[26] A.J.Sachs & H.Hunger, Astronomical and Related Texts from Babylonia Vol I, p.351, recorded 'Night of the 19th, last part of the night, Mars was 1 ½ above α Virginis'.
[27] Longitude and Latitude of star and planet assumed unchanged over short difference in time

Appendix B - Earlier Developments in the Region.

1. Horizon Alignments.

The supposed temple at Tell es-Sawwan is an early example, from the middle of the 6th millennium BC, of a building oriented about 45° from the cardinal points.[28] Later at Teleilat Ghassul (level IV) in Palestine there is a remarkable wall painting of an eight-pointed star.from about –4000.[29]

Other bearings are evident at Nabta Playa and Eridu.

Nabta Playa.

At Nabta Playa there are alignments of megaliths radiating around a central point. Their bearings are in three bands A (26/31°), B (117/122°) and C (127/131°).[30] In turn these can be divided into narrower ranges, but here we will look at the positions of the individual stones as, with a fixed central point, it only takes one marker to define an alignment.

For each megalith, the differences in longitude (converted to great circle degrees) and latitude, from the central point, were divided by a factor.[31] A unit of 0.00194°, corresponding to c. 215 metres, gave significant results for bands A and B.[32] Of the 17 positions no less than 8 had longitudes (expressed in linear units) equating to either whole or half units. Of the radii from the central point 8 equated to either whole or half units. This cannot be accidental. It would appear that they were determining positions by any two of the following: the radius from the centre and the easterly or northern component from the centre. In other words any two sides of a right-angled triangle.

Band C does not fit this analysis, which is not surprising as Malville et al concluded it was 'problematic because of migration of the stones.[33] However one stone (C5) is still of interest, as its unit co-ordinates are 2.7 (S) and 3.6 (E) and with a radius of 4.5 units from the centre. The alignment corresponds to the hypotenuse of a Pythagorean triangle with sides in the ratio 3, 4 & 5. In this case the unit would be 193.5m.(10% smaller than mentioned above). There is the possibility that one of the attractions for the placing of the central point (A) was its position relative to C5, which was described as a 'dispersed cluster of blocks' with a large original size of 'about 2.0 x 1.5 x 0.3 m.'[34]

Bands A and B are largely confined to two 3.4° segments, between bearings determined by the ratio ½, the tangent of 26.6° and the sine of 30°, measured from due North (A) or

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[28] Edwards I.E.S et al, The Cambridge Ancient History, Vol.1, Part 1, 1980, p.274, Fig. 21.
[29] Edwards, op. cit, Vol. IV, p.522 and plate 14c.
[30] Malville J.M. et al, Astronomy of Nabta Playa, in Holbrook J. et al, African Cultural Astronomy, Springer 2008, p.137.
[31] Brophy T.G and Rosen P.A, Satellite Imagery Measures of the Astronomically Aligned Megaliths at Nabta Playa, Mediterranean Archaeology and Archaeometry, 2005, Vol.5, No.1, pp15-24, Table 1.
[32] The calculation is based on a great circle degree of 111 km. According to Petrie (Encyclopaedia Britannica 1951), the Egyptians had a khet (100 cubits) with a length of 52.37m. 4 khets would be 210m. Subdivisions smaller than a half, were probably tenths rather that quarters or thirds. There is some indication that the unit length rose from about 211m in –4400 to 218m in –3600. If we assume that the three A positions A1, A2 & A3 were all intended to be 4 units north and 2 units east of the centre, the units would range from 0.00185 to 0.00195°.
[33] Malville op. cit, p.139
[34] Wendorf F. and Malville J.M, The Megalithic Alignments in Wendorf F. and Schild R, The Archaeology of Nabta Playa, 2001, p.494.

due East (B). The two segments are 90° apart. The bearing of the rising sun at the winter solstice would have been 26.1°, south of due east, in -4700

From the table below, we can see that the rising of Sirius, the brightest star in the sky, would have aligned with the stones in the B band from about –4700 to –3700, but with a gap from –4200 of nearly 400 years.[35] An adjustment of nearly 2° was then needed.

Similarly the rising of Arcturus would have matched the megaliths in the A band from –4450 to –3600, but with the largest gap from –4275 to –4100. To put these dates in perspective, it is thought the Egyptian Civil calendar with 365 days in the year was established around –4500.[36]

Table 3. Individual Megaliths at Nabta Playa

Ref	Size	Position		Difference		Linear measures			
		Lat.	Long	Lat	Long	Lat	Long	Radius	Year BC
	Cu.m.	Degrees	degrees	Degrees x 100	Gt. Circle degrees x 100	units	Units	units	
Centre A		22.5080	30.7257						Arcturus
A2	3.7	22.5157	30.7298	0.77	0.38	**4.0**	**2.0**	4.4	4450
A3	0.7	22.5155	30.7297	0.75	0.37	3.9	1.9	4.3	4430
A1	2.9	22.5158	30.7299	0.78	0.39	**4.0**	**2.0**	**4.5**	4400
A0	0.4	22.5136	30.7288	0.56	0.29	2.9	**1.5**	3.2	4275
A4	1.4	22.5149	30.7297	0.69	0.37	3.6	1.9	**4.0**	4100
AX	0.4	22.5164	30.7306	0.84	0.45	4.3	2.3	4.9	4075
A5	1.4	22.5131	30.7288	0.51	0.29	2.6	**1.5**	**3.0**	3920
A6	?	22.5135	30.7291	0.55	0.31	2.8	1.6	3.3	3850
A7	0.5	22.5131	30.7289	0.51	0.30	2.6	**1.5**	**3.0**	3800
A8	1.0	22.5127	30.7287	0.47	0.28	2.4	1.4	2.8	3720
A9	1.0	22.5121	30.7284	0.41	0.25	2.1	1.3	**2.5**	3600
									Sirius
B7	0.5?	22.5065	30.7283	-0.15	0.24	-0.8	1.2	**1.5**	4700
B6	0.1	22.5063	30.7288	-0.17	0.29	-0.9	**1.5**	1.7	4460
B5	?	22.5061	30.7293	-0.19	0.33	**-1.0**	1.7	**2.0**	4200
B3	5.2	22.5059	30.7300	-0.21	0.40	-1.1	**2.0**	2.3	3820
B1	?	22.5058	30.7303	-0.22	0.42	-1.1	2.2	**2.5**	3750
B4	?	22.5060	30.7299	-0.20	0.39	**-1.0**	**2.0**	2.3	3700
C5	0.9?	22.5027	30.7333	-0.53	0.70	-2.7	3.6	**4.5**	

This analysis indicates that 12 of the 17 megaliths in bands A and B were placed in three short periods of greater activity: –4450/ –4400 (4), -4100/-4075 (2)–3850/-3700 (6). Only two were placed in the 300 years from –4400 to -4100 (exclusive), which matches the three centuries, when the lowest number of samples were found for radiocarbon dating (Figure 8B). We can perhaps see this period as being one of low human activity in the area and is consistent with the megaliths in the A & B bands being placed individually to point to the rising of Arcturus or Sirius.

The megaliths would also align with other less bright stars. For example Sirius and α Centaurus rose at the same point on the horizon around –4400 and thereafter markers which had served previously for Sirius would serve for α Centaurus, as it moved lower in the sky.

[35] Star data from SkyMap Lite 2005.
[36] Wells R.A. in Walker C. (Editor), Astronomy before the Telescope, British Museum, 1996, p.34

The distance from the central point would vary as they sought integer values for linear measurements of any two of the radius, latitude or longitude, to determine the precise position

In general they seem to have been less tolerant of imprecision in the case of Arcturus than Sirius. Consequently there are more alignments for the former, possibly because the slow movement northwards of the rising of Sirius was already well known. Unlike Sirius, the rising of Arcturus was moving southwards, which may have attracted closer attention.[37] The first four alignments for Arcturus are near 26.6°, with a tangent of 0.5. The difference in bearing for these four was less than one degree, which suggests an aim for high precision.

Eridu.

At Eridu not all the many levels of temple construction were perfectly rectangular and the early walls varied significantly in bearing.[38] At Napta Playa the lines of stones, radiating around a centre, point solely to the eastern horizon, but at Eridu the walls can be seen as aligned between opposite points on the western and eastern horizons (Table 4).

Table 4. Walls at Eridu

Level		Walls					Stars			
		SE	NW	NE	SW		α CMa	α Cen	α Lyr	κ Ori
		Bearings - degrees				Year	Longitude/Horizon Azimuth-degrees			
18		30/210[39]	30/210							
17		30/210	29/209	126/306	127/307	-5100	8/**127**			
16		30/210	30/210	126/306	126/306	-4900	10/**126**		189/25	
15		35/215	39/219	130/310	130/310	-4700	13/125	155/121	192/26	354/**129**
11		37/217	37/217	127/307	127/307	-4250[40]		161/124	199/29	0/**126**
9		37/217	37/217	127/307	127/307	-3750		167/**127**	205/32	7/122
8		40/220	41/221	132/312	133/311	-3000		177/**132**	210/36	
7		40/220	40/220	131/311	131/311		See footnote 31			
6		53/233	53/233	143/323	143/323	-2700		181/226	220/**323**	
Level 6 excluded		Corresponding Declinations Degrees								
18/7	rising	48/41	49/41	-30/-35	-30/-36					
18/7	setting	-48/-41	-49/-41	**30/35**	**30/36**		Zigpu stars in bold (on left)			

We can distinguish four distinct groupings:

1. In each of the first three levels, 18/16, there is at least one wall oriented 30°/210°. This suggests a subdivision of the horizon into 30° segments, with the two middle segments, totalling 60° in the east and west, corresponding to slightly more than the annual range of the sun at the horizon.[41] The 30°/210° alignment would complete the 30° segments and would mark the centres of the two bands, which the sun does not

[37] The rising of Arcturus moved 5° southwards in 850 years and of Sirius 4.7° northwards in 1000 years

[38] The alignments were taken from Edwards I.E.W., Gadd C.J., Hammond N.G.L. (Editors), The Cambridge Ancient History, CUP, 1980, Figures 24 & 25, pp 335 & 338. Figure 24 shows levels 18 to 8 and although small has the advantage of having just one indication of north for all levels. In figure 25, the other two levels, 7 & 6, each have their own north pointer. In this analysis level 7 with an indicated date around –3100 would be out of sequence with level 8.

[39] Three of the walls are aligned 30/210°, while one, the most northerly, is about 29/209°

[40] α Cma and α Cen would have had the same declination c. –4400, which falls between levels 15 and 11.

[41] In the middle of the 6 millennium BC, with the obliquity of the ecliptic 24.2°, the theoretical range would be 29.4° either side of due east/west.

reach and which are not circumpolar. The divisions between the major 60° segments lie either side of an alignment 60/240° or 120/300°.

2 In levels 11 and 9 the buildings are more closely rectangular and oriented in accordance with the angles in the simplest Pythagorean triangle, with sides in the ratio 3,4,5.[42] One wall at level 17 is similarly aligned.[43] The same angles are also evident in the last level (6) but transposed. Six of the nine identified levels had walls in this or the previous group.

3 Excluding levels 18 and 6, the remaining seven have at least one wall on a bearing of 126/132° in the east and 306/312° in the west. These two ranges correspond to objects with complementary declinations of –30/-35° and +30/+35°, either rising in the east or setting in the west. The latter range would include what were later termed zigpu stars, which transit overhead and ideally had a declination of 30.5° at Eridu.[44] The former range would, at different times, have included two of the brightest stars as at Nabta Playa.

Of the five brightest stars Canopus (α Car) and Arcturus would have been too low or too high, leaving Sirius, α Cen and Vega (α Lyr). The brightest star, Sirius, would have risen on a bearing of 127° in –5100 and 126° in –4900, when it would have been opposite, in longitude, to Vega and so six months apart. As Sirius rose, Vega was 21° above the western horizon. Later Sirius became too high, but α Cen would have been in range (levels 11, 9 & 8). This leaves a gap between levels 15 and 11, which could have been filled by a star of Orion, such as Saiph (k Ori), which, although not particularly bright, is part of a very obvious constellation and was also opposite the sun at the autumn equinox. An alternative would have been the brighter Rigel (β Ori)

With levels 17 and 16 two hundred years apart, we might estimate the date of level 18 as about –5300. Overall the range would be from then until level 6 in –2700. Postgate gives a range from c.-5000 to c.-3000.[45] Bienkowski and Millard give a span of 'at least 1500 years from 5500 BC or earlier'.[46] The dates suggested here, although not coincident, are similar to those indicated by these two authorities. We can probably have the greatest confidence in those for levels 17 and 16, associated with the rising of Sirius, levels 11 or 9 and 8 associated with the rising of Rigel Kentaurus and level 6, associated with the setting of Vega.[47]

Egyptian 5-pointed star.

In the coffin lid tables (see below) the epagomenal stars are grouped together, but we should not rule out the possibility that at some earlier stage a single day was inserted into the calendar every 72 days.

The star hieroglyph with five spokes, implying the division of a circle into 72° segments, is known from the earliest Dynasties.[48] It is not the easiest form to draw, so there must have been a good reason for its adoption.[49] It is shown with one spoke vertical and the others on either side at angles of 72° and 144°. Table 5 gives details of five stars which, at

[42] The angles are 36.9° and 53.1°
[43] It is possible that there may have been some lack of differentiation between the various levels.
[44] Hunger H and Pingree D, MUL-APIN, An Astronomical Compendium in Cuneiform, Archiv fur Orientforschung, Horn, Austria,1989, pp 141/4.
[45] Postgate J.N., Early Mesopotamia, Routledge, London, 1996, p.25 caption to figure 2:2.
[46] Bienkowski P and Millard A. Dictionary of the Ancient Near East, British Museum, London, 2000, p.107.
[47] Other than for level 6, Vega seems to have been consistently mis-aligned by about 4/9°.
[48] Petrie H, Egyptian Hieroglyphs of the First and Second Dynasties, Quaritch, London, 1927.

Abydos around –3900, would have, almost simultaneously, been on the horizon. The two rising stars, λ Tel and 110 Her, are not particularly bright.

Table 5. Calculated for –3900 at Abydos (Lat. 26.2°)

Star	Magnitude	R.A.	Declination	Horizon Azimuth	Diff Azimuth
α UMi	1.86	322	58	343	71
γ Gem	1.93	19	0	271	72
α Car	-0.62	66	-58	199	71
λ Tel	4.85	184	-34	128	71
110 Her	4.19	220	32	54	75

It would not have taken long to realise that α UMi spent about one fifth of a day below the horizon and was separated from β UMa by a similar length of time. These two northern stars would have facilitated the visual subdivision of the area around the pole into five equal segments.

Therefore a plausible alternative justification for the hieroglyph would be that the spokes are separated by 72° in time. α UMi with a declination of 58.7° would rise and set 36° (time) from lower transit and 144° from upper transit. It would be 72° below the horizon between setting and rising, which would be 36° apart in azimuth.

α Umi would have had such a declination around –3900 and at the same time it and β UMa would have been 72° apart (R.A.). Other stars with about the same declination would have been γ Dra and one of those in the Corona Borealis constellation. As α Umi set, α Car was also setting, which provides additional support for the five-pointed star being related to the rotation of α Umi around the pole.[50] The suggested date of –3900 is commensurate with the –4500 given by Wells for the determination of the length of the year as 365 days.[51]

Table 6. Calculated for –3900 at Abydos (Lat. 26.2°)

Star	Magnitude	R.A	Difference	Declination	Horizon Azimuth	Time to nearest transit	Long approx
		degrees	degrees	degrees	degrees	degrees	degrees
α UMi	1.86	322	83	58	17	35	7
β UMa	2.34	34	72	62	8	17	57
α2 CVn	2.84	94	60	64	n.a.	n.a	
α CrB	2.21	167	72	56	21	42	140
γ Dra	2.24	240	73	61	12	5	186

A few decades later α UMi would spend 69° below the horizon, which would match the 70 days spent in the duat, which traditionally is associated with the time that Sirius (α CMa) is too near the sun to be visible. Maybe there was more than one manifestation of the 70 days in the duat.

By this time the five-pointed star might have come to represent the daily passage of time, around the pole. The rising and setting of a Umi (R.A.322.5) could have served as a

[49] Roaf. M, Cultural Atlas of Mesopotamia, Equinox, Oxford, 1990, p.70. shows a pictographic sign for a star with eight spokes around –3100.
[50] At this time α Umi and α Car would have set 143° apart in azimuth.
[51] Wells R.A., op.cit. p.34.

control, with the other four stars being a Uma (25), e Uma (105), y CrB (171) and e Dra (253.5). The successive differences in R.A (in brackets). range from 62° to 83°, so would not have been at all precise.

In the Pyramid texts, the word for hours is determined by three stars.[52] Sticking with α Umi, the other two could have been β Cva (86) and η Her (202.5). The differences in R.A would be 116.5°, 120° and 123.5° and, if correct, would indicate much greater precision. This is speculative, but seeks to explain how the measurement of time could have reached the high level of precision built into Kafre's and later pyramids (see below).

There is further reference to a five-pointed star below under Hierakonpolis.

Alignment of Mastabas at Saqqara

The northerly alignments of all but one of the long sides of the mastabas of the 1st Dynasty at Saqqara are in one of two groups 330/341° and 355/358°.[53] The first is roughly parallel to the Nile, which along this stretch flows towards 335°.

With this relationship to the river, it would not have gone unnoticed that around –2920, when α UMi was at upper transit, the setting of Corona Borealis was aligned with the river (Table 7). We see that constellation as a crown, but then it might have been likened to a bowl or the sign N41/42, a 'well full of water'.[54] On setting its 'rim' would have been level with the horizon on a bearing between 331/341°, matching the first of the two groups of mastaba alignments.[55] At the same time α CMa would have been 2° below the horizon and about to rise.

Table 7. Data for –2920 on a latitude of 30°, α Umi at upper transit

Star	Magnitude	R.A.	Decl	Horizon Azimuth	Altitude
		degrees	degrees	degrees	degrees
ι CrB	4.98	190	52	335	0.1
ε CrB	4.14	188	49	331	-3.5
γ CrB	3.8	183	49	331	-5.0
α CrB	2.21	180	51	333	-4.9
β CrB	3.65	179	53	337	-3.0
θ CrB	4.06	181	55	341	0.1
α UMi	1.86	330	63	N/a	57.0
α CMa	-1.44	47	-22	115	-2.0

[52] Clagett M, op.cit. Vol.II p.49. He presumes that this was linked to the Civil Calendar with 12 months in three seasons.
[53] One mastaba is aligned 10° west of north, about midway between the two groups.
[54] Gardiner Sir A., Egyptian Grammar, Oxford University Press, 3rd Edition, 1969, p. 492.
[55] Over the years in question, precession would not have played a significant role in the spread of the mastaba alignments.

2. A Portable Sketch from Saqqara – Pythagorean triangles and a spiral.

From Dynasty 3 (c.2600 BC), we have a sketch of an arc, which Marshall Clagett described as ‘a kind of descriptive geometry born of practical measurement…’.[56] There may be rather more to it than that.

The crucial unknown is the distance, assumed to be equal, between the Y ordinates. Clagett followed Wolff in taking it to be 28 digits or 1 Royal cubit. However, if it was actually 24 digits, the co-ordinates would be 0,98, 24,95, 48,84, 72,68 and 96,41.[57] The sketch then incorporates three Pythagorean triangles, with their long sides parallel to the X axis (Figure 9):

14, 48, 50 (7,24,25) linking points 1 and 3,
54, 72, 90 (3,4,5) linking points 2 and 5,[58]
30, 72, 78 (5,12,13) linking points 1 and 4.

The coordinates (96,41) of the fifth point suggest that there was a fourth triangle with sides 9,40,41 below it.

An Egyptian architect with Pythagorean set squares could delineate curves in integer rectangular co-ordinates, which a builder could readily follow. In this example the architect drew a rough arc on a piece of limestone, to which he added his previously calculated Y ordinates.

But what was the curve he had in mind? Points 1,3,4 & 5 lie close to a circle, but its centre (–10,-30) is well away from the vertical axis through point 1, and point 2 does not fit.

Two other possibilities are:

1.The curve is an approximate protractor for angles 15°, 30°, 45° and 67.5°.

2.The curve is part of a similar spiral to that used at Babylon, where the X co-ordinate is proportional to the angle below the horizontal at point 1 (see Table 8).[59] With the exception of point 3, the others are close to a ratio of 7.5° per cubit of 24 digits. This value, known as a part, or 48th of a circle, belongs ‘to an early sequence of primitive angular measures’, according to Neugebauer.[60]

The 3,4,5 triangle for points 2 and 5 fits the second alternative better than the first. (see last column in Table 8).

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[56] Marshall Clagett, Ancient Egyptian Science, Vol. III, 1999, pp. 78/79, 109 note 68 and 462. The curve is not a single circular arc as the radius for the points 1, 2 & 3 is less than that for points 3, 4 & 5.
[57] The Egyptian short cubit contained 6 palms and 24 digits.
[58] The 3,4,5 and 5,12,13 triangles intersect at 45,79.25 and 60,68. The 11 digits just below point 2 are divided precisely into 4,3,4 digits. The triangle of 3,4,5 digits would be, in palms, ¾, 1. 1 ¼, which is similar to how it appeared in the very much later Baylonian tablet Plimpton 322 (see below).
[59] This is a similar arrangement to that at Babylon for measuring azimuth, where the ratio was 2.5° per cubit.
[60] Neugebauer. O., A History of Ancient Mathematical Astronomy, Springer-Verlag, 1975, Part Two, p.671.

Table 8. Analysis of Five Points in Sketch

Point	X	Y	Angle from Vertical at origin 0,0	Assumed Target	Difference	Angle below horizontal at point 1	Divide X by 3.2	Difference
	digits	digits	Degrees	degrees	degrees	degrees	digits	degrees
1	0	98	0	0	0	0	0	0
2	24	95	14.18	15	-0.82	7.1	7.5	**-0.4**
3	48	84	**29.74**	**30**	**-0.26**	16.3	15	+1.3
4	72	68	46.64	45	+1.64	**22.6**	**22.5**	**+0.1**
5	96	41	66.87	67.5	-0.63	30.7	30	**+0.7**

Spirals are known in Egypt from the 1st and 2nd Dynasties, so it is worth examining how they might relate to the Horus Eye Fractions, 1/64, 1/32, 1/16, 1/8, 1/4, 1/2, which were represented by parts of an eye and used for volumes of grain.[61] The first quadrant of an Archimedian spiral would have an area of just under 32 square units, if the radius for the 90° position was 11 units.[62] This area would split into two halves along a line parallel to the short axis, 5 units from the origin and a similar line 11/10 units from the origin would delineate 1/16th of the total area (Figure 9b). Between these two lines there would remain 7/16ths, of which 1/4 would be represented by a circle with a radius of 1.6 units.[63] The form of the individual fractions are roughly similar to the ancient glyphs, except for 1/8 and 1/32 above and below the circle respectively.

Problem 10 in the Moscow Mathematical Papyrus refers to a basket with an area of 32 and to this being half an egg shell.[64] It seems therefore that the egg consisted of the two initial counter-rotating quadrants of Archimedian spirals, so that in terms of area an eye was half an egg, divided lengthwise and a basket was also half an egg, presumably with the egg divided at right angles.[65] Evidently the units were not the same!

The basket also had an opening, presumably a diameter, of 4.5 units, which using the Eyptian method of calculation would have an area of 16 sq.units, so was twice the size of the 1/4 Horus eye fraction and consequently was 1/4 of the egg. If the opening had a depth, rather than a diameter, of 4.5 units then it would equate to the distance from the origin to where the egg was widest.

3. **Hierakonpolis, Pyramids and Horizontal Hour-lines.**

It will be shown below that, by the Pyramid age, they had mastered the use of horizontal dials to measure time. At Hierakonpolis they may have already started on that long journey of discovery.

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[61]Petrie H, Egyptian Hieroglyphs of the First and Second Dynasties, Quaritch, London 1927. Plate XXXVi shows 5 spirals (855/859) from the Royal tombs and Hierokonpolis. They rotate both clockwise and anticlockwise, so could readily have been put together to form an egg. She also showed eyes (plate III) with one (64) enclosing a circle.

[62]The area of the first quadrant is $\pi n^{2/}/12$, where n is chosen as 11. This is the smallest integer value that has another integer, 5, for the division of the total area into halves and another integer, 3, for the distance from the origin to the centre of the circle. It could be readily rescaled.

[63]In the drawing it is shown as touching the lower edge of the eye, but there ought to be a small gap to allow the two areas either side to count together as 1/32.

[64]Gillings R.J., Mathematics in the Time of he Pharaohs, Dover, 1972, p 195

[65]Divided along a line parallel to the short axis produces two very different halves – one pointed and the other rounder. In this regard we should remember there were other similar Egyptian signs which look similar to eggs and/or baskets.

Hierakonpolis Sites HK29A & HK29B and the Narmer Palette.

Although the two adjacent sites are of a similar age, HK29B will be looked at first as it appears more experimental and less sophisticated than HK29A.

HK29B

The layout of HK29B reflects at least three variations of position and height of the gnomon; the western (green) and eastern (red) palisades and the radiating post holes (blue), shown in figure 8c2.[66]

The obliquity of the ecliptic was falling slowly from 24.1° in -4000 to 24.0° in -3000. At the winter solstice, the sun's declination would have been c.-24.1° in the first half of the 4th millennium. At night someone following a star with the same declination would traverse the same path as the sun's shadow of the top of the mast.[67]

Green Layout.

The line of the palisade, rather than being a smooth curve, is more like two straight lines, and this is brought out more clearly in Aziz et al where the further westward extension to the palisade shows up as a third straight section, offset slightly to the north.[68] With a gnomon of 5.5m in the green position, the shadow of the rising mid-winter sun would be just south of the line of the western palisade, which hints at its purpose.[69] To plot the path of a star, like the shadow cast by the sun, an observer ought to get his eye down to ground level, but it would be less awkward if he could track the star along the top of a low fence. However this would create its own problem: the paths at the top and bottom of the fence would not represent the same line of constant declination. A fence of 80cms in the green scheme would reduce the effective height of the gnomon from 5.5m to 4.7m. and the result can be seen in figure 8c2. The dotted green line represents a constant declination of -25.1° on top of the fence: to the west it is further north and on the meridian further south than that for -24.1°, at ground level. Along the palisade, in the middle section, they are almost indistinguishable. That vertical section of palisade would represent declinations in the range -24.1°/-25.1°. Sirius (α CMa) in -3680 had a declination of -25.1°, so could be taken as having a similar path to that of the sun at the winter solstice.

Red Layout.

The eastern section of palisade (red) is less straightforward. The western end is aligned with the rising of the mid-winter sun, with a gnomon of 4.0m in the position shown, and is sensibly parallel to the western (green) palisade. However the eastern part has a significant

[66]Friedman R., Hierakonpolis Locality HK29A: The Predynastic Ceremonial Centre Revisited, JARCE Vol.45, 2009, pp79/103. I am most grateful to Dr. Friedman for sending me a copy of her detailed paper, which permitted a closer analysis of the first two phases. The drawings are based on her figures 1, 8 and 9.
[67]Most of the relevant observations were well away from the horizon and above an altitude of 6° where atmospheric refraction is only about 0.14°. Consequently its effect on declination is small and has generally been ignored.
[68]Aziz A.M., et al, Remote Sensing at Hierakonpolis, Nekhen News 30, page 25. It seems they preferred successive straight lines to curves.
[69]The green palisade is now known to extend in the same direction for about 50m, further than shown in figure 82c2 (Private communication from Dr. Friedman)

change of direction and also, unlike the green palisade, continues further eastwards until it reaches the main (green & blue) meridian, but without showing the curvature there that one would expect.[70] With a gnomon of just 1.4m this part of the palisade would match the 'shadow' of a star with a declination of -29.6° (figure 8c2).

The green and red layouts mark the transition from simply watching the shadows on the ground to observing them in the vertical plane of a palisade.

Blue Layout.

This more sophisticated layout is presumably later than the green or red.[71] The principal line with eight post holes cuts the main meridian, 8.55m north of the gnomon (altitude 29.2°, declination -35.7°) at the same post hole as the red line for -24.1°.[72] [73]

The details of the most westerly 7 post holes of the principal line are set out in Table 9, where the most westerly and easterly have been numbered 14 and 7 respectively.[74] They lie on a straight line, aligned with the point near the equator and east of the gnomon, where a large post hole is marked by a blue cross. This point is 2.24m north and 3.56m east of the gnomon and would be on the equator, if the gnomon height was 4.78m.[75]

From the fifth column we can see that the radius from the gnomon is 135 units for position 14 and reduces in approximately 10 unit steps to 67 units for position 7.

The seventh column includes three simple tangent ratios for the altitude of positions 14, 11 & 7 and combining them with the distances of the post holes from the gnomon, we can calculate the height of the gnomon as 4.795m, 4.771m and 4,764m, averaging 4.78m.

The estimated declination of about -35.7° is shown in the final column. This is for 'shadows' on the ground, but we cannot rule out the possibility that the observer also climbed the posts to view nearer the horizon.

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[70] At its northern end the main meridian has a large circular structure with a diameter of c. 1.6m,. It is mentioned by Hikade but without any indication of its function (Hikade T.. Origins of Monumental Architecture: Recent Excavations at Hierakonpolis HK29B and HK25), Might it perhaps have served as a podium for an observer watching the successive transit of stars across that meridian?

[71] In the red scheme the post holes are little more than 60cms apart, in the green it is slightly more and in the blue over 3m, along the principal -35.7° curve. Previously the use of 'a block with, in the centre, a vertical pole of eye height' was suggested as a moveable marker for celestial objects. (Bremner R.W., The Shadow Length Table in mul-Apin) Graz,1991, published in Galter H.D., Die Rolle der Astronomie, 1993, pp 367/382. Hikade op.cit. mentions carved figures. Putting the two together perhaps the 'blue' holes are for fixed carved markers.

[72] The hole on the meridian for a declination of -35.7 would be 9/5 of the gnomon height from its base.

[73] If this was insufficient justification for a post hole on the meridian, then it may also have served for yet another gnomon for an unidentified fourth layout.

[74] With this numbering there appears to be a missing post at number 8.

[75] The ratio of 2.24 and 4.78 is 0.47 and corresponds to the tangent of the latitude of Hierakonpolis 25.1°. Expressed in units of 0.32m the position of the large post hole is 7 units north and 11.1 units east of the gnomon.

Table 9. Post Holes along principal line (-35.7° Decl), Gnomon Height 4.78m (14.94 units of 0.32 m) No allowance for refraction.

No.	West	North	Radius	Radius in units of 0.32m	Apparent Altitude	Simple Tangent ratio (altitude)	Estimated height gnomon	Declination
	m.	m.	m.	units	degrees		m	degrees
14	-30.79	30.24	43.16	134.9	6.36	1/9 (6.34°)	4.795	-35.7
13	-28.31	28.18	39.94	124.8	6.86			-35.6
12	-26.07	26.21	36.97	115.4	7.41			-35.6
11	-23.22	24.00	33.40	104.4	8.19	1/7 (8.13°)	4.771	-35.5
10	-20.96	22.17	30.50	95.3	8.95			-35.9
9	-18.35	20.25	27.33	85.4	9.97			-35.9
7	-13.79	16.41	21.44	67.0	12.64	2/9 (12.53°)	4.764	-35.8

The seven post holes were carefully positioned along the straight line, with the most westerly three, particularly no. 13, are almost precisely 45° (azimuth) from the meridian. The tangents of the altitudes of the most westerly and easterly are 1/9 (6.34°) & 2/9 (12.53°).

Radiating lines, dashed blue, around the post hole, marked by a blue cross, match the near straight sections of constant declination, -38.3° and -31.7° and their inclinations to the equator are 42.25° and 34.3°, with tangents 0.91 and 0.68. These inclinations and modern declinations are linearly related and, given that the middle inclination for -35.7° is close to 4/5, it is possible the two outer ones were intended to be 9/10 (41.99°) and 7/10 (34.99°).

The blue cross marks the point around which the 'shadows' of stars, with differing declinations, appear to rotate for part of their length.

If, at this stage, they had recognised the 'pole' on the ground, it would have been 10.2m to the south, but that area has not been excavated and, in any event, there does not appear to be evidence of an interest in measurements around the pole at this stage.[76]

They were well able to manipulate angles using ratios of their tangents, much as a roofer does to-day[77]. They also seem to have found a system for delineating lines of constant declination on the ground, albeit only over a limited range (-31.7°/-38.3°) and altitudes (6.3°/12.5°).

A lower altitude limit of 6.34°, on the principal line, avoided the highest values for refraction close to the horizon, but nevertheless it would still have had a small measurable effect. We can be fairly sure of the intended altitudes for positions 14, 11 and 7, namely 6.34°, 8.13° and 12.53°. At those apparent altitudes, the true altitudes would be closer to 6.21°, 8.02° and 12.47°. Starting with an initial estimate of 35.7° for declination, we can

[76] A possible exception is the principal blue line for -35.7° declination, where the most westerly post hole is 61°, a little over 4 hours, from transit. The first seven post holes cover 10° or 40 minutes of time.

[77] Draper J.T., The Steel Square applied to Roof Construction, The Technical Press Ltd., London, 1930. For 116 different angles between 29.92° and 78.68° the author gives the tangent ratios, which an ancient Egyptian would have readily understood, once he had mastered 1/16ths of an inch.

calculate, marginally different, estimates for the true declination (-35.78°, -35.77° & -35.74°, averaging -35.76°) for the three positions with simple ratios for altitude.[78]
With so many variables, including the height and position of the gnomon, it is impossible to be sure which stars might have justified the choice of the declinations identified. However their range is limited to -24.1°/-38.3° and it is quite likely the latter was chosen because its path would cross the meridian at a point twice the height of the gnomon from its base and not for the brightness of any particular star.[79] This reduces the range to -24.1°/-35.7°, containing four of the brightest stars (Table 10). Firstly we should note how little the declinations changed over 150 years, so even if we could confidently identify the star, it would not closely define a date. All that can be said is that at some point in the 150 years after -3700 each of these bright stars would have matched an identified declination, but it does not prove that this was when attention became focused on that particular declination.

Table 10. Bright Stars within the band of declinations in the years -3700/-3550. Data from StarMap Lite 2005.

Scheme & Gnomon Height	Declination Tan(Meridian Altitude) & respective Declination	Bright Star/ Magnitude	Star Declination with observation at ground level		
			-3700	-3600	-3550
	Degrees		Degrees	Degrees	Degrees
Red palisade 1.4m	-29.6 7/10 (-29.9)	β Ori/ 0.17	**-29.60**	-29.04	-28.77
Green palisade 5.5m	-24.1/-25.1	α CMa/ -1.44	**-25.18**	-24.77	-24.58
Blue - Southern 4.78m	-31.7 13/20 (-31.9)	α Cent/ -0.01	**-31.58**	-32.07	-32.2
Blue - Main line 4.78m	-35.76 5/9 (-35.8)	α Crux/ 0.77	<u>-35.27</u>	<u>-35.61</u>	<u>-35.79</u>
		eps CMa/ 1.5	<u>-35.95</u>	<u>-35.57</u>	<u>-35.41</u>
	Star with near constant declination	TYC 8169 1192 1 3.75	<u>-35.54</u>	<u>-35.52</u>	<u>-35.53</u>

Hierakonpolis is on a latitude of about 25.1° the tangent of which, 0.4684, can be considered as either 7/15 (0.46) or a more precise 15/32 (0.4688), so we might expect the height of the gnomon to be divisible by either 15 or 32. The estimated height of the gnomon is 4.78m, which divided by 15 indicates a unit of 0.319m.

This analysis assumes the shadows were observed at ground level, but the observer could perhaps have climbed the vertical poles to get observations nearer the horizon, but this would have increased the influence of refraction. Assuming a perfectly horizontal horizon

[78]These calculations ignore temperature and barometric pressure, so give only an approximate idea of the true declination, which is required if one is aiming to determine the year a particular star reached that declination.
[79]At a distance of 10 units a mast of 5 units would subtend an altitude angle of 26.57° and on a latitude of 25.1° this would correspond, on the meridian, to a body with a declination of 38.33°.A similar value seems to define the northern boundary at HK29A.

and maximum refraction of 34 minutes, it would push the lower limit of declination to c.-40°.

HK29A

Friedman divides the development of HK29A into three phases and here we will look at the first two.

Phase 1 (c.-3500).

This phase is focused on a much wider range of declinations and specifically on the area of the 'pole' on the ground. Because some of the site has still to be excavated, we have to deduce the positions of certain key elements, including the pole, indirectly.

The most southerly two rows of four post holes (a/d and e/h) are respectively aligned 12.65° and 18.3° from east/west (Fig. 8d2). The angles correspond to the smallest angle in a Pythagorean triangle with sides in the proportion 9, 40 & 41 (12.7°) and to an angle, whose tangent is 1/3 (18.4°). The two lines converge to a point a little over 10m south of the mast (feature 16).[80]

If the 9,40,41 triangle was scaled up to 18,80,82, then the unit of measurement would have been about 324mm and the radius from the point of convergence for each of the four post holes (a/d) would be 82, 68, 57 and 47 integer units or 'feet'.[81] These radii are the same for three (f/h) post holes in the second row. The short sides of the Pythagorean triangle and of its mirror image are shown dashed in figure 8d, with the short side of the mirror image passing close to the most westerly post hole (e) in the second row. The long side of the mirror image aligns approximately with the most westerly of the northern post holes (i), so the line from the point of convergence is inclined about 25.4° to the east/west line.

Two holes (m & n) are aligned with the 'shadow' of a body with a declination of just over 38°.[82] The point of convergence of the two lines would correspond to the 'pole', if the mast was 4.8m tall and was used as a gnomon.[83] The line of the equator would be 2.25m north of the mast.[84] Any 'shadows' of stars which crossed the two most southerly lines of posts together would transit together across the meridian. The most northerly posts (i/l) are not well aligned in this phase. The two western post pits (i & j) and the most easterly (k & l) lie north and south, respectively, of the east/west line through the mast, indicating greater

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[80]The two lines converge on a point about 10.25m south of the mast (feature 16). The latitude of HK29A is 25.1°, whose tangent is 0.4684, almost exactly 15/32., which would suggest that they were using a linear unit of about 320mm (10.25/32), so that the height of the mast was about 4.8m or 15 units. This agrees with Friedman's description of a 'tall solitary pole...'.

[81]This estimate of 324mm is 4mm greater than that deduced previously (footnote 80). Based on the radii, we can calculate the West and North coordinates from the pole as 80,18 - 66.3,14.9 - 55.6,12.5 - 45.9,10.3 units of 324mm.

[82]By 'shadow' of a star is meant the position of an observer's eye when using the top of the gnomon as a foresight to follow a star. The rising sun would cast an elongated shadow of the mast and the top would briefly appear on any vertical fence or wall around the courtyard. If the top of the mast carried an emblem, such as a hawk or scorpion, its shadow would appear on the fence or wall, before sliding down into the courtyard.

[83] The distance of the equator from the mast, 2.25m, is equivalent to 6.9 or 7.0 units for a unit of 324 or 320mm.

[84]The distance being (height of mast x 15/32). If the height was 15 units (see footnote 80) the distance would be $15^2/32$ or 7.03 units.

interest in that line than in the equator. There are several stars with declinations that would align with the four post holes, but β1 Sco is notable because it would match position j and three other nearby stars would align with i, k and l at the same time (see Table 11).

In phase 1 much of the northern side of the courtyard is bordered by the gateway and Wall Trench 1. The path of a star with a declination of -38° would cross the gateway, more or less tangentially .[85] We have to consider the possibility that Wall Trench 1 was intended for a wall that would serve to raise the level for observations to the same as at the base of the mast, although if that was so, we would expect a change in direction at the meridian whereas, in fact, it continues until the end of the courtyard.

Table 11 Star data for -3500 from SkyMap Lite 2005 (all values in degrees).

Position	Altitude	Decl.	Time to transit	Possible Star /magnitude	Altitude star	Decl. star	Time to transit star
i	11.3	6.7	80.7	ν Sco/4.0	11.6	6.6	80.4
j	13.0	6.6	78.7	β1Sco/2.56	13.0	6.6	78.8
k	16.1	9.0	76.3	θ Lib/4.13	16.4	10.0	76.5
l	19.6	12.3	74.0	γ Lib/3.91	21.0	12.9	72.6

The coherence of so many positions suggests that they were indeed using the mast as a gnomon. On that basis we can calculate the paths of celestial bodies with various declinations (blue in Fig. 8d2), assuming the courtyard was sensibly level with the base of the mast. Any unevenness in the ground surface would divert the paths of constant declination from the calculated curves.

We have seen above that for the two hour-lines radiating around the pole (a/d and e/h), they showed a preference for angles, which were either part of a Pythagorean triangle or had other simple trigonometrical ratios. Therefore in Figure 8d the hour-lines are taken from Table 12.[86]

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[85] Other parts of Wall trench 1 would match more closely the path of a body with a declination slightly above -38°. In -3400 α1 Crux had a declination of -36.3°.

[86] Th calculation is Tan (Hour-line angle) = Sin (latitude) x Tan(Time to meridian)

Table 12 Hour-lines with simple trigonometrical ratios (angles and time in degrees)

Hour-line angle to the meridian	Tangent, Cosine (C) or Pythagorean triangle	Time to meridian	Comments
26.56	1/2	49.7	Western gate post & Star Decl. -38°, post holes m & n aligned with declination path +38°.[87] A star with a declination of -38.3° would cross the meridian at an altitude of 26.6° (same as Hour-line angle), so its horizontal distance to the base of the mast would be twice the height of the mast.
36.87	3,4,5	60.5	Path Decl. -30° enters courtyard at salient point; Post hole d aligned with declination path +30°
49.4	7/6	70	Mid winter sun & Sirius enter courtyard, declination -24°[88]; bodies with declinations between11° & 24° enter courtyard
53.13	3,4,5	72.4	Time to meridian compatible with five pointed star (72°).[89] Hour-line touches perimeter of courtyard in several places, so includes a wide band of declinations. Declination +20° is close to post hole l and altitude on meridian 45°, so distance from base of mast equals its height.
60.0	(C) 1/2	76.2	Hour-line tangential to perimeter of courtyard between equator and east/west line through mast

Points X & Y are 45° from the meridian, measured around the pole and mast respectively. They correspond to bodies with declinations of -27.3° and -35.0° and on the perimeter of the courtyard are 66.8° and 58.4° (time) from the meridian.

Phase 2. c. -3300.

On the northern side, the perimeter of the courtyard is now further south and towards the east is bordered by Wall 1, which partially coincides with the path of a body with a declination of c. -35° (Fig. 8e).[90] Such a body would no longer cross the meridian twice the height of the mast from its base, but would be at an altitude closer to 30°, so the distance to the top, not the base, of the mast would be twice its height.

On the south, outside the courtyard, the realigned northernmost line of large post pits is c.23° to due east/west, which is comparable to the smaller angle in a Pythagorean triangle with sides in the ratio 5,12,13 (22.6°), scaled up by 6 to 30,72,78, with the four posts 30,

[87]East of the gateway the path for a declination of -38° closely matches the line of Wall Trench 1, which suggests the possibility that the latter was the trench, not for a wall, but for a pathway raised to bring it to the same level as the base of the mast, although in that case one might have expected it to stop or change direction at the meridian.

[88]Sirius and the mid-winter sun would have had similar declinations of c.-24.1° c. -3410 (Data from SkyMap Lite 2005), but were well apart in R.A. (c.132°). This convergence in declination was probably also noticed at Newgrange in Ireland. O'Kelly C., Concise Guide to Newgrange, Houston, Cork, 2003 and see www.Mythical ireland.com. There is a book (not seen) The Newgrange Sirius Mystery by E.A. James Swagger.

[89]Petrie H., Egyptian Hieroglyphs of the First and Second Dynasties, Quaritch, London 1927, p.xxi, figures 490/492, from the Royal Tombs.

[90] Both ε CMa and α Lep had declinations of about -34.5° in -3300.

24, 20 & 16 integer units (323mm), north of the pole.[91] The line cuts the meridian about 7.6m south of the mast, indicating that in this phase the pole was nearer the mast, which was lower at c.3.56m. A little south of the four posts there is an almost parallel trench.

The constant declination paths for ±24° enter the courtyard 72° (time) from the meridian. For the mid-winter sun it is at the western apex and in mid-summer it is just below the 'll' of the 'Wall 2' label. Between these two extremes the hour-line for 72° runs roughly parallel to the longer arm of the brickwork, near the western apex, and then crosses the equator at the most easterly of the three indentations in the courtyard perimeter.[92] Evidently this indicator of time was of considerable importance and furthermore it would have been consistent with drawing stars with five points. It seems fairly certain that, by this stage, they were thinking in terms of six hours of 72°, with an average 'hour' of 12°. They were also exhibiting a closer interest in the line of the equator. There the 78° hour-line would be tangential to the perimeter, and 6° from the 72° line, a difference of half an 'hour' of 12°. The linear distance between the two would be about 6.56m or 20 units of 328mm. The westernmost indentation would correspond to the 75° hour-line from the meridian and from Figure 8e, we can see that the hour-lines for 60° and 45° corresponded to the north-eastern side of the platform and to westernmost end of the perimeter, where it abuts Wall 1. Even if an average hour of 12° was top of their thinking they could also accommodate one of 15°. At the time of the pyramids, there was a standard hour (15°), measured along the equator (see below).

Table 13 summarises the various hour-lines and the points to which they are aligned. It would seem that by Phase 2, they were able to handle more sophisticated tangent ratios than in Phase 1.

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[91]The size of the unit (323mm) agrees closely with that deduced for phase 1 (324mm), but slightly less well with the 320mm, which may indicate a slight error in the estimated positions of the pole. The calculated Western ordinates from the pole would be 72, 57.6, 48 & 38.4 units of 323mm.

[92]The 72° hour-line was closely bordered on the south by a protuberance on the brickwork near the western apex. Presumably this was to set the southern limit for this hour-line and might have been aligned with the appearance of the 'shadow' of Sirius. However the declination of Sirius was gradually rising and so would soon have rendered such a marker obsolete, possibly justifying the subsequent removal of all the brickwork.

Table 13 Hour-line angles and time to meridian, using 17/40 for sine(latitude), correct for 25.15°

Desired Time to Meridian	Calculated Hour-line angle to meridian	Approximate Tangent ratio	Time to meridian, using tangent ratio	Diff.	Alignments[93]
degrees	degrees		degrees	degrees	
45	22.99	17/40	45	0	western end of abutment of courtyard & Wall 1
60	36.31	80/109	59.93	-0.07	north-east corner of platform
64	41.01	20/23	63.95	-0.05	north-west corner of platform
67	44.98	1/1	66.97	-0.03	perimeter west of platform & 45° from meridian
68	46.40	20/19	68.01	+0.01	salient point on courtyard perimeter
72	52.55	80/61	72.04	+0.04	centre western apex, -24° shadow enters courtyard and eastern indentation on equator
73	54.22	40/29	72.87	-0.13	aligned to northern side of brickwork and eastern indentation on equator
75	57.72	8/5	75.12	+0.12	aligns with several points on perimeter and western indentation on equator
78	63.39	2/1	78.00	0	tangential to perimeter on equator
80	67.47	12/5	79.96	-0.04	four post-holes

We have noted above that they appeared to have used linear units in the range 320/324mm. The values differ by little more than 1%, which may be due either to errors in our measurements or that, at the time, each 'surveyor', had his own set of measuring equipment and there was little desire for a single uniform standard.

On the south, outside the courtyard, the realigned line of large post holes is c.23.1° to due east/west and cuts the meridian about 7.6m south of the mast indicating that in this phase the pole was nearer the mast and the mast was lower at c.3.56m

With the two alignments in the same area from Phase 1 we can see how they were handling angles (Table 14).

[93]For hour-lines 75° and below, there are small indentations along the southern perimeter of the courtyard, although they may not all be significant. The two small indentations in the perimeter, north and south of the mast, presumably marked the meridian.

Table 14

Phase/row of posts	Approx. inclination to east/west	Rounded Tangent (degrees)[94]	Pythagorean Triangle	Time to transit
	degrees			degrees
1/southern	12.7	9/40 (12.7)	9,40,41	85
1/middle row	18.5	1/3 (18.4)		82
2/wall trench 2	20.5	3/8 (20.6)		81
2/ northern	23.1	5/12 (22.6)[95]	5,12,13	80

It seems clear that they were using simple ratios of the sides of right-angled triangles to define angles around the pole. We can therefore assume that similar 'nice' angles would be used for other divisions crossing the courtyard. Five are shown in Figure 8e and in Table 15, of which three are related to the line of the equator, which as shown below, was of particular interest at the time of the pyramids.

Table 15

Hour-line angle to meridian	Rounded Tangent (degrees)	Pythagorean Triangles	Time to transit	Comments
36.9	3/4	3,4,5	60.5	Northernmost point of platform
45	1/1		67.0	Western boundary of Pit Dynasty 1
53.1	4/3	3,4,5	72.4	Western end of Courtyard & on eastern indentation of courtyard perimeter on Equator
60	[96]		76.2	Nearly tangential to courtyard perimeter & on western indentation of perimeter on Equator
63.4	2/1		78.0	Tangential to courtyard perimeter on Equator

For an hour-line angle of 53.1°, the time to the meridian is a little over 72° or 1/5th of 360°, which makes it compatible with a five-pointed star.

In phases 1 & 2, the height of the mast appears to have been about 4.8m and 3.6m respectively, but the midwinter sun would still have risen on the same bearing and consequently the shift in where the shadow of the mast entered the courtyard would only have changed by about 60cm. Consequently in the plans of the western apex, the outline does not change between phases 1 and 2.

Although the outline here is the same, there were significant changes within the perimeter. In the early phase there was a cluster of post holes, which by the second phase had been

[94]The Egyptians are known to have used the inverted tangent or seked, which can be seen as the length of the sun's shadow, using a gnomon of unit height.
[95]It could also have been 90/4 or 22.5°
[96]60° angle equals the angle of each segment of a hexagon or it can be considered as 90°-30°. The sine of 30° is 1/2, which is thought to have been of interest to the builders of the alignments at Nabta Playa.

replaced by a thick wall, facing to between the mast and the pole. The southern end of this wall turned to run irregularly towards the pole, before apparently stopping just before reaching an unexcavated area. This is in contrast to the short section of wall next to the platform, which faces towards the eastern end of the line of large postholes, south of the courtyard. Whatever its purpose it was surely not the same as the other pair of walls, the longer of which runs towards the pole with a small roughly northern accretion, which stops short of the line between the pole and the point where the shadow of the top of the mast enters the courtyard in mid winter, leaving a clear line of sight between the two. The two walls may have served partly to modify the ground level in the area, possibly bringing it up to the same level as at the base of the mast or higher.

The middle of the wall at the far western end of the courtyard, aligned c.55° from the meridian, faced the theoretical rising point of the mid-winter sun. If someone climbed the mast and sat on the top, he would have seen something remarkable: his life-sized shadow 40m away on the vertical face of the wall, at the end of the much elongated shadow of the mast.[97] This near miracle was perhaps the inspiration for the figures of men bearing extremely long flagpoles, known from the Narmer palette and elsewhere.[98]

Narmer Palette[99],

This magnificent ornamental palette was found at Hierakonpolis and has had various interpretations with the smiting figure often being associated with Orion. For the reasons set out above, the presence of the four men bearing long flagpoles also implies a celestial source. The side with the circular grinding area could represent the northern sky, towards the end of the 4th millennium B.C., with Auriga top centre, near the zenith, and flanked by Gemini and Perseus as heads of cows. The long-necked serpopards, around the pole, would be based on Ursa Major, Ursa Minor and Draco with their feet on the horizon, below which there is the figure of a bull breaking into a partially walled city, corresponding to Corona Borealis.[100] The bull, the northern part of Hercules, appears to be pushing the stars around the pole and above the horizon. The tops of the four long standards might be associated with Lynx.

At the same moment, the other side of the palette would represent the southern sky with Orion in the position of the smiting figure, wearing the crown of Upper Egypt, and Taurus as the hawk. The same three constellations remain at the top in an east/west line, but now in reverse order.

If correct the Narmer palette would be the earliest known attempt at depicting parts of the sky, running here from below the horizon in the north, through the zenith, to below the horizon in the south.

Strabo attributes the invention of the science of geometry to the Egyptians and from what we have seen above they were already well versed in this subject.[101] Even without the concept of 'angles' they had reached an astonishing level of competence. They were

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[97]There would be some slight distortion in his shadow if the wall was not perpendicular to the light of the sun.

[98]Emery W.B., Archaic Egypt, Penguin, 1984, pp 44/5. He also illustrates the mace-head of the Scorpion king with dead birds hanging from masts and the mace-head of Narmer with his standard bearers.

[99]This palette, found in 1897/8, is now in the Cairo museum, but pictures are readily available on line.

[100]The gap in the city walls corresponds to the gap between i & θ Corona Borealis.

[101]Strabo, The Geography of Strabo, Translated by Jones, H.L. William Heinemann, London, 1930, Vol. VII, pp 269/270.

conversant with the role of the pole, although they do not seem to have adopted a fixed 'hour' measure, whether of 10°, 12° or 15°, all of which were tried later (see below).

Pyramids and Horizontal Dials

In the Pyramid Texts, on the walls of 5th and 6th dynasty pyramids, Utterance 251 includes ‘O you who are over the hours…..’ and Utterance 320 ‘The King has cleared the night, the King has despatched the hours….’.[102] ‘In both passages the word for hours (*wnwt*) is determined by three stars, suggesting to us that the most primitive meaning of “hours” was “nighttime hours”.[103] The more precise measurement of time by the stars was clearly established by the 5th Dynasty (2500/2350 B.C.. We will see that they were interested in the line of the equator at an early date, presumably sparked by the almost perfectly straight line, from west to east, of the shadow of the sun at the equinoxes. When time-keeping by the stars became more important it would have been convenient to have had plenty of stars close to the equator and the number peaked around -2300 (Table 16).

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[102]Faulkner R.O., The Ancient Pyramid Texts, OUP, 1969. The earliest surviving example is in the pyramid of Unas (2373 BC) bur no single pyramid contains the whole text.

[103]Clagett M., op.cit. Vol.II American Philosophical Society, Philadelphia, 1995, p.49. The Egyptians employed a year of three seasons aligned with the rise and fall of the Nile, so it is tempting to assume the three stars were separated by 120°. This may not be correct as there is another small group of three (α Libra, β1 Scorpio, ε Ophiuchus) each separated by close to 15° (R.A), ideal for establishing an hour standard.

Table 16. Stars with magnitude <5 near equator[104]

Star	RA	Magnitude							
	-2300		-2500	-2450	-2400	-2350	-2300	-2250	-2200
			Declination - Arc Minutes						
δ3 Taurus	0.6	4.29							-2
ε Taurus	0.6	3.53		-2	15				
11 Orion	18.7	4.65						-12	4
15 Orion	19.8	4.81				-11	4		
134 Taurus	29.8	4.89			-5	10			
28 Monoceros	65.4	4.68						-7	0
TYC 4857 2151 1	72.1	3.91					-12	-7	-2
27 Hydra	86.8	4.80							-12
υ2 Hydra	98.4	4.60						12	9
y Hydra	145.2	2.99				5	-9		
β1 Scorpius	184.0	2.56				9	-7		
ω2 Scorpius	184.1	4.31	-4						
ω1 Scorpius	184.1	3.93	6	-11					
ν Scorpius	185.6	4.00				9	-8		
ψ Ophiuchus	188.2	4.48	-11						
χ Ophiuchus	189.2	4.18				17	0		
TYC 6221 904 1	192.8	4.91				4	-12		
η Ophiuchus	200.0	2.43				2	-14		
o Serpens	208.1	4.26					7	-8	
ξ Scutum	219.0	4.66					7	-6	
α Scutum	222.1	3.85					4	-9	
η Scutum	227.9	4.83					7	-14	
12 Aquila	229.1	4.02		9	-3				
λ Aquila	230.4	3.43				5	-6		
ι Aquila	238.8	4.36						6	-3
η Andromeda	320.3	4.40	-3	10					
λ Aries	334.3	4.79			-6	8			
α Aries	336.3	2.01			-14	1			
ξ Aries	352.1	4.00							-8
Number of Stars			4	4	5	11	13	9	8
Closest pair or 0			-3/+6	-2/+9	-3/+15	-11/+1	0	-6/+6	0
Balanced pair			-4/+6	-11/+10	-15/+14	-11/+10	-7/+7	-6/+6	-3/+4

Figure 9c with four pyramids attributed to Sneferu (3) and to his son Khufu (1) indicate that they were conversant with the design of a horizontal dial with, by the time of the Great Pyramid, half-hour divisions. Earlier the 45° hour-line, passed under the centre of the satellite pyramid and met the equator on the enclosure wall at Meidum, while at the North or Red pyramid it was the 48° hour-line. There the northern and southern corners of the pyramid coincided with 36° and 72° hour-lines, suggesting that they were experimenting with an 'hour' of 12°. The 30° hour-lines met the equator as it crossed the base of the Bent pyramid.[105] Curiously, in plan view, the bend on the southern side of the Bent pyramid coincided with the 30° hour-lines, in a similar manner to the upper missing part of the Meidum pyramid.[106]

The inclination of the plane of the equator would have been observable where the shadow of the sun crossed an enclosure wall at the equinoxes. Along the meridian it would be possible to measure the distance of the equator from the centre of a gnomon and from its known height calculate the length of the meridian in the equatorial plane and from that length deduce the distances along the equator for any time from the meridian.[107] Drawing lines from those points to the pole would give the respective hour-lines.

[104] Data from SkyMap Lite 2005

[105] Although these points influenced the design of the complex, observations could not actually be made at many of them. For example at the Bent pyramid an observer at a corner of the base could not see the apex.

[106] At Meidum the pavement surrounding the enclosure wall suggests that this pyramid may have ben given over to the study of the sky. If this was so, could it have been partly dismantled to discover what would happen with a 'bent' pyramid?

[107] The calculation being Length of the meridian in the equatorial plane x Tan (Time from meridian)

For the range of geographic latitudes of the pyramids, we would to-day happily assume they were all on 30° when setting up a garden sun-dial, but the size of the pyramids makes this questionable. Table 17 shows the distances along the equator for various times to the meridian. For times of 60° or more there are noticeable differences in the cubit values, although using the same values for all pyramids would not introduce a major error in time measurement.[108]

The final column in Table 17 shows the rounded distance in units of 28.87 cubits, which as shown below seems to have been used for some aspects of the layout of the pyramid complex of Pepi II.

Table 17. Distances, in cubits, along the equator for successive half-hours from transit, assuming a pyramid with a height of 100 cubits (52.5m) and geographic latitudes of Meidum and 30°, near Abu Roasch.

Latitude(°) Tangent Sine	29.388 0.5632 0.4907	30.000 0.5774 0.5000	30.000 cubits/28.87	30.000 rounded units of 28.87 cubits
Time from meridian (°)	cubits	cubits	units	units
7.5	15.110	15.202	0.527	$^{15}/_{28}$
15	30.752	30.940	1.072	$^{15}/_{14}$
22.5	47.539	47.829	1.657	$1^2/_3$
30	66.262	66.667	2.309	2⅓
37.5	88.065	88.603	3.069	3
45[109]	114.769	115.47	**4.000**	**4**
52.5	149.570	150.484	5.212	5¼
60	198.785	200.000	6.928	7
67.5	277.077	278.769	9.627	$9^2/_3$
75	428.323	430.940	14.927	15
82.5	871.756	877.082	30.288	30
Equator distance	56.320	57.735	**2.000**	**2**
Pole distance	177.558	173.205	**6.000**	**6**

The pyramid of Khufu shows their mastery of horizontal dialling, with the Queens' pyramids and several boat pits positioned in close relationship to hour-lines, in half-hour divisions.[110] The northernmost boat pit in the north-east is close to the junction of the equator and the circle around the pyramid, where the altitude of the sun or star would equal the slope of the corners. This is an early intimation that this altitude was of particular interest.

Figure 9d has another four pyramids, with Khafre's, Menkaure's and Sahure's demonstrating an interest in the area to the east of the pyramid. In Khafre's the times for a

[108]Using 428.33 cubits along the equatorial line (correct for 75° to the meridian on the latitude of Meidum) on a latitude of 30° would give a time of 74.91°, a difference of 22 seconds.
[109]The cubit values are the same as the length of the meridian in the equatorial plane.
[110]The Queens' pyramids were positioned in relation to the centre on triangles with sides in the ratios 11,60,61, 1,2,√5 & 3,4,5, scaled up so that the long side was 195 cubits or 102.4 m, using 525mm for the length of a cubit. This compares with 524mm, deduced by Petrie W.M.F,, Encyclopaedia Britannica , 1951, Vol. 15, p.144.

celestial object on the equator to reach the altitudes of the corners and the sides would be 2.5 and 1.5 hours from the meridian. That of Userkaf does not follow this trend to the east with most ancillary structures being around the pole, but it is the first of the pyramids with a standard slope of 53.13°.[111] Its enclosure wall on the east is close to the pyramid, but with room for the point where the altitude, along the equator, is the same as the corners of the pyramid, marking one end of the standard hour. The other end of this standard hour was within the pyramid and therefore inaccessible. However it would be clearly signalled by the disappearance of the shadow of the pyramid. At Menkaure's and Sahure's this point on the equator also coincided with an azimuth of 45°, which too would be visible outside the pyramid.

The pyramid of Sahure has an enclosure wall indented beside the equator and the lengthening evening shadow would run from 36° to 53°, a range of 17°, with room for a standard hour now completely outside the pyramid.[112] The same general layout to the north-east of pyramids endured for some 550 years until Senworset I.

Figure 9e shows another four pyramids, all with a similar layout around the north-east corner and their details are summarised in table 18, which also includes those of Kafre, Sahure, Pepi II (ignoring the girdle around the base) and the much later Senworset I (-1965/-1920).

Table 18.

Pyramid	Times for shadow leaving the pyramid and meeting the enclosure wall along the equator and the range	Times for shadow leaving the pyramid and meeting the enclosure wall on the North and the range	Times to transit when altitude the same as the slope of the corners/ sides	Difference from one hour
	degrees	degrees	degrees	seconds
Kafre	33/n.a	33/n.a	22.48/37.58	+23
Sahure	36/53 - 17	36/42 - 6	27.61/41.73	-211
Djedkare	33/49 - 16	33/45 - 12	22.72/37.73	+1
Unas	31/50 - 19	31/46 - 15	16.35/32.95	+385
Teti	33/52 - 19	33/48 - 15	22.69/37.71	+5
Pepi I	34/51 - 17	34/47 - 13	22.72/37.72	+1
Pepi II[113]	33/51 - 18	33/45 - 12	22.74/37.74	-1
Senworset I	37/52 - 15	37/46 - 9	29.2/42.98	-294

[111]Over the following 300 years the same slope was used in another six pyramids, if one includes that of Djedkare-Isesi. which was originally omitted because of small inconsistencies in its dimensions and the slope of its sides. Its design is important because it incorporated the standard slope of 53.13° (as in a 3,4,5 triangle) and a height of 52.5m (100 cubits) with a layout, around the north-east corner, similar to that of Sahure. Five of the standard pyramids had the same dimensions, a height of 100 cubits and a base side of 300 cubits.

[112]What drove their interest in an 'hour standard', which can be traced to the pyramids of Khafre and Menkaure? Although they could accurately calculate time along the equator, they would not get precisely the same results, in practice, if two pyramids were significantly apart in longitude. At Giza, the three pyramids have longitudes: Khufu (31.1342°), Kafre (31.1308°) and Menkaure (31.1283°), a range of 21 seconds. This might have been a source of frustration, in the absence of any notion of a spherical earth. An 'hour standard' would have had almost precisely the same length in cubits for all three pyramids, after due allowance for their different heights.

[113]The measurements ignore the girdle around the base of the pyramid, the effect of which is discussed below. With the girdle, the time on the equator, at ground level, would be reduced from 18° to 14°, insufficient for an hour.

For seven pyramids, from Sahure's to Senworset's, the layout in the north-east corner could accommodate an hour standard along the equator, but only in those of Unas and Teti could the same hour be measured along the northern enclosure wall.[114]

Previously it was noted that the standard pyramids had a built-in standard hour along the equator, when the celestial body was at altitudes equalling the slopes of the sides and corners.[115] This was part inside and part outside the pyramid. The details are summarised in the last two columns in the table above. This characteristic was the result of the adoption of the standard pyramid, with a slope of 53.13°, which, if built on a latitude near 29.845°, would create this very accurate standard hour.[116] It had ceased to be of importance by the time of Senworset I.

Pyramid of Pepi II

In the last and most southerly, that of Pepi II, we can 'see the plan of the standard pyramid complex in its final and most developed form'.[117] However it had an added girdle, 6.5 metres (12.4 cubits) in width, which increased the sides of the base from 150 to 174.8 cubits.[118] Consequently, although included in table 18, because of the added girdle around the base, a standard hour of 15° could not be accommodated at ground level (fig 9a).
In relation to the main pyramid, those of the wives were positioned using Pythagorean triangles (Table 19 and Figure 9a).[119]

Table 19. Pepi II - Pyramids of wives

Pyramid	Centre Measured on plan		Pythagorean Triangle & (scaling)	Calculated centre	
	West	North		West	North
	cubits	cubits		cubits	cubits
Iput II	160.8	119.1	3,4,5 (40)	160	120
Neith	66.4	155.8	5,12,13 (13)	65	156
Wedjbeten	79.4	152.4	8,15,17 (10)	-80	-150

On the other hand, some distances in the Pyramid complex appear to be based on a standard, related to the distance of the equinoctial line from the centre (Table 20).

[114] It is assumed that all measurements started at the time the equatorial shadow appeared out of the pyramid.
[115] Bremner R.W., Letter to BAA Journal, Vol. 127-1, February 2017, page 55. D. Rawlins pointed out the inconsistent dates in that original letter.
[116] The calculations both at the time and now ignore the effects of geocentric parallax.
[117] I.E.S. Edwards, The Pyramids of Egypt, Penguin, 1993, p.181.
[118] Edwards, op.cit. p.188. The girdle may have been required for reinforcement. We do not know its height, but the width was 6.5m or 12.4 cubits. On the small plan the side measured 172.5 cubits, compared with the calculated value of 174.8 cubits, a difference of ca. 1.3%. This gives a rough idea of the precision of the measurements.
[119] With the four already noted above (p.15), two at Khufu's and one at Pepi II's, brings the total of different Pythagorean triangles to 6: 3,4,5-5,12,13-7,24,25-8/15/17-9/40/41-11/60/61.

Table 20. Pepi II - Grid of c. 28.8 cubits

	Distance cubits	Divisor	Unit of Measurement cubits
Eastern Wall of Mortuary Temple to centre pyramid	259.8	9	28.9
Girdle side	172.5	6	28.7
Satellite Pyramid N	114.7	4	28.7
Satellite Pyramid E	72	2½	28.8
Diagonal Open court	58.1	2	29.0
Diagonal of Iput II	56.7	2	28.4
Equinoctial shadow line	57.36	2	28.7
Distance on equator between 35° & 50° & between 40° & 50°	56.7 40.7	2 √2	28.3 28.8

This suggests that a unit of about 28.8 cubits was used for some aspects of the layout, with 12.5 of these units being 360 cubits. Today we might be tempted to think of it as 90/ π (28.6) and a circle with this radius would have a circumference of 180 cubits with a ratio of 2° per cubit, one of the ancient norms.[120] Then it may well have been derived from 50/√3 (28.87), which agrees with the calculations in Table 20.[121]

In reality none of the standard 3,4,5 pyramids is precisely on a latitude of 30° and therefore their values are slightly different.

The equatorial line coincides with the northern wall of the mortuary temple, but the hour standard, identified in table 11, cannot be accommodated, as already mentioned. On the other hand, before the addition of the girdle around the base of the pyramid, an hour, from 35° and 50° after transit, would fit neatly within the open area immediately north of the sanctuary. In that position it would serve for objects in the western sky, using the apex as a foresight. In the narrow gap between the pyramid and the enclosure wall in the west, a star could only be observed close to 35° before transit (ignoring the girdle).[122] In the pyramid of Pepi II, the girdle would reduce the level area along the equatorial line. It is suggested that, to overcome this setback, they opted for a short hour of 10° or 40 minutes. For 40° and 50° from transit the observer would be, respectively, 96.7 and 137.4 cubits from the meridian, with the difference being close to an average of 4 cubits per 1° of time or 1 cubit per minute.[123] A short hour of 10° appeared later in the diagonal star tables on coffin lids.[124]

The sanctuary would restrict observations of stars above the equator, but to the south the absolute limit would be –18° declination on the meridian.[125] Away from the meridian such a body could only be observed from outside the enclosure wall. Within it and north of the

[120] 90/π equals 28.8, if π is taken to be 25/8. Intriguingly at just over 72° from the meridian, the distance is 360 cubits and 1st, 2nd & 5th Dynasty representations of stars show them with five points (see footnote 48).

[121] The later Shadow Clock, described in the Cenotaph of Seti 1 is different, as it appears to use hours of 60 minutes. (Clagett op. cit. pp.463/470 has a translation).

[122] In round numbers stars on the equator could not be observed within 35° of the meridian, mimicking the 70 days passed in the 'duat'. See Clagett op.cit p. 364/5, referring to the Book of Nut.

[123] Expressed in units of 28.87 the two values in cubits become 3.34 and 4.76 The ratio would be exactly 1 cubit per minute, on average, between 39.5° and 49.5° from transit, with the distances from the meridian being 95.0 and 135.0 cubits.

[124] Wells R.A., op.cit., pp 37/8. The earliest of these tables date from the 9th Dynasty, soon after the reign of Pepi II.

[125] The declination of Sirius would only have risen to –18° by 1425 BC.

pyramid the declination would be around –12°, which, crossing the meridian 90 cubits north of the apex, avoids the girdle and allows observation along the length of the enclosure wall. Significantly the causeway for Khafre's pyramid had an azimuth, directed at the rising of a body with a more precisely defined declination of –11.8°.[126] The sun would have such a declination two months from the winter solstice and would delimit a season of the four months with the 120 shortest days.[127] A calendar for an Egyptian year of three seasons could thus be kept in step with the sun, with the other two seasons being either side of the summer solstice.

At night, four bright stars were in the band between 0° and –12° of declination (Table 21). Sirius itself was too low, but the Sothis constellation included her head-dress, so δ Monoceros, with a similar R.A., is taken as the exemplary star. One of the 36 ten-day decans is 'Red One of Khenett', identified as the red α Scorpio (Antares, Rival of Mars). Between it and δ Monoceros there were 136 days and 13 decans, which are sufficiently correlated to justify the identifications.

Table 21. Bright Stars with declinations between 0° & -12°

Star[128]	Magnitude	Equatorial Co-ordinates -2300		Julian Day Re-based	Diff R.A.	T class 10 day decans [129]	
		R.A.	Decl.	days	degrees	No.	Re-based
α Taurus	0.75	11.7	-1.1	-179	-176	24?	-18?
y Orion	1.64	26.3	-7.5	-164	-161	26?	-16?
α Orion	0	33.0	-4.1	-157	-155	27?	-15?
δ Monoceros	4.15	53.8	-4.3	-136	-134	29	-13
α Scorpio	0.88	187.8	-7.8	0	0	6	0

These stars are close to the equator, where we have seen that the distance between 40° and 50° from transit is just over 40 cubits and for bodies with a declination of –12° it would be 38 cubits between 30° and 40°.[130] In both cases it would average about 1 cubit per minute. The Egyptians were clearly able to measure time (months, days and hours) rather better than is usually acknowledged.

4. Coffin Lid Tables in Egypt.

Two centuries after the building of the last standard pyramid we have the first coffin lids with astronomical tables. These tables list 36 decan stars, at 10-day intervals, plus 5 epagomenal days, in accordance with the Egyptian calendar. We have seen above that they were using an hour of 60 minutes, but the girdle added to the pyramid of Pepi II, may have forced them to employ a shorter hour of 40 minutes. They would then have had to rework

[126] Nell, E. and Ruggles C.,The Orientations of the Giza Pyramids and associated structures, University of Leicester, version 2 – 15th March 2013, p.37, Table 12.
[127] The sun's R.A. being 208° & 332° with a difference of 124°.
[128] Star data from StarMap Lite 2005.
[129] S. Symons, S, A Star's Year in J.M. Steele (editor), Calendars and Years, Oxbow, 2007. p.8 (Table 5). Decans 24 and 25 refer to the Arm [of Orion]. In the K class (Table 6) the difference, between Sothis and the Red One, is 14, not 13, decans.
[130] For comparison, the distance for the original hour standard was 41 cubits for 60 minutes. The distances along the equatorial line being 48.3 and 89.2 cubits for altitudes of 53.13° and 43.314°.

earlier schemes and, on this basis, we suggest dating the surviving coffin lid tables to about –2250.

The majority of the coffin lids of known provenance come from Asyut on a latitude of 27.23°, which has certain interesting properties. The sun at the solstices would rise 27.16° from due east, which is almost identical to the altitude of the pole.[131] Less obviously the azimuth, swept by the sun at the two extremes, would be 153° and 207°, if measured from rising to 270°, which closely matches the time of the sun above the horizon, 154° and 206°. A change of 1° azimuth corresponded, on average, to 1° time above the horizon. They had the means to measure time for celestial bodies with declinations between +/-30°.

Table 22. Asyut - Latitude 27.23° Rising Azimuth and Time above Horizon

Rising Azimuth from North	Declination	Azimuth swept to 270°	Time above horizon
Degrees	degrees	degrees	degrees
50	34.9	220	222
60	26.4	210	210
70	17.7	200	199
80	8.9	190	189
90	0	180	180
100	-8.9	170	171
110	-17.7	160	161
120	-26.4	150	150
130	-34.9	140	138

Symons allocates the 19 known coffin lid tables to one of two classes K(7) and T(12), in which the Sothis constellation, with Sirius, is placed 36th and 29th respectively. The five epagomenal days follow the 36th decan, so were respectively either 10/15 or 80/85 days after Sirius.[132] In what follows we will examine the epagomenal stars in the K class.[133]

The possible concept behind the scheme is that for 360 days there was a selection of 36 stars which progressed through 360° in R.A. but only 355° of longitude.[134] In the next five days longitude would reach 360°, but R.A. would change very little.

[131] Assuming 23.95° for the obliquity of the ecliptic.
[132] Locher has identified the sceptre of Sothis on a coffin lid as representing a line of stars from β Col to η Lep, which implies a year beginning, not ending, with Sirius – see Von Bomhard A-S, The Egyptian Calendar, Periplus, London 1999, p. 23, Fig. 17. Possibly the image represents another tradition.
[133] Between Crux and Corvus there are many stars where those for the epagomenal days in the J class might be found.
[134] If R.A. and longitude had the same value, the stars would lie on a circle mid-way between the ecliptic and the equator with their declinations and latitudes having the same absolute value but with the opposite sign.

Table 23. Possible Epagomenal Stars (a/e) with First and Last Decan Stars for –2250
(Last Column – azimuth differences with δ Cma on meridian and 5 days later)

Star	Number K class	Mag.	R.A.	Decl.	Long.	Lat.	Day	Azimuth from meridian	
			degrees	degrees	degrees	degrees		degrees	
α Cma	36	-1.44	55	-20	46	-39	0		
Calculated									
Ideal	a		65	-26	56	-47	10		
Ideal	1		65	-8	61	-29	15		
Possible Stars								meridian	+5 days
δ Cma	a	1.83	65	-28	55	-49	10	0	5.1
TYC6537	b	4.83	65	-25	57	-46	11	-0.5	5.0
TYC5974	c	4.94	64	-20	57	-41	12	0.3	6.3
FW CMa	d	4.14	64	-17	57	-38	13	1.2	7.6
KQ Pup	e	4.82	65	-15	60	-36	14	-0.7	6.1
α Mon	1	3.94	65	-10	61	-31	15	-0.3	7.3

At the same time of day, the five day change in azimuth is between 5° (δ Cma) and 8° (α Mon), while their R.A. is sensibly the same. By having five epagomenal stars instead of a single half-decan, the adjustment is spread over five days, which suggests daily time-keeping was of paramount importance, but they could tolerate a daily adjustment of little more than 1°. Could they have tried to accomplish this by using offset meridian lines, for the five epagomenal stars? The daily offset would have been successive one-fifths of the overall adjustment. In practice this is not straightforward with these actual stars.

It is easy to calculate the R.A. of each of the 36 decan stars, but without being able to pinpoint their declinations, although –30° would be attractive.[135]As they were evidently prepared to use relatively faint stars, it is not difficult to suggest one for each of the 36 decans. Although, even with a sizeable population to choose from there must have been the odd one which did not fit the scheme precisely. For example with three adjacent stars with 9° and 11° (R.A.) between them, the outer two could be timed on the meridian, but the middle one would be 1° out. To overcome this, they might well have used a pseudo–meridian, one degree offset from the true meridian, for just that one star. Subsequently this could have developed into a grid to cover the area around the meridian, such as can be seen in the Ramesside Star clock of ca. -1470.[136]

At first sight such observations were made by one of two observers, seated facing each other, with the horizontal positions of stars indicated by parts of the other observer's body, such as his eye, ear or shoulder. Neugebauer describes the method as 'incredibly crude'.[137] Perhaps the second observer was only to be imagined, rather as we visualise a clock when indicating directions by the position of an imaginary hour hand. When my oculist says look at my ear, he wants me to look in the direction of his ear, not study it!

From at least the Old Kingdom, Egyptian artists used square grids to set out human figures.[138] It would not be a big step to use parts of the human body to indicate a particular

[135] Multply class K row number by 10 and subtract 305 to get R.A. in –2250. A star with a declination of around –30°, near the meridian, would move 10° in azimuth over 10° time.
[136] Clagett op. cit. Vol II p.406.
[137] Neugebauer O., op…cit p.561.
[138] Robins G, Proportion and Style in Ancient Egyptian Art, Thames & Hudson, London, 1994, p.59

gridline with eye, ear and heart representing the three successive lines from the centre. What angles might have been represented? The proportional distances are in the ratio of about 1, 3 and 6, so if the first line was at one degree, the others would have been about 3° and 6° from the centre.

5. Pythagorean Triangles and ratios of angles, including time, to linear units.

In the Old Babylonian period (ca. 1800 BC), they were well versed in Pythagorean triangles. The Ark tablet contains a value, 14430, for the necessary rope and this can be expressed as 2 x 3 x 5 x 13 x 37, where the last three factors equal the hypotenuse of a Pythagorean triangle.[139] A figure of 2405 (5 x 13 x 37) contains the hypotenuse of no less than 13 Pythagorean triangles – 5, 13, 37, 65(2), 185(2), 481(2) & 2405(4). A circle with such a radius has 108 points with integer co-ordinates, including the four cardinal points.

The more famous tablet, Plimpton 322, has 15 extant rows, each referring to a Pythagorean triangle, although some have argued that the scribe intended to complete a total of 38 rows, covering the edge and both sides of the tablet.[140] There may be good reasons why he stopped at the 15th row.

The tablet is broken and the rows are incomplete, but it is believed they would have included, in two missing columns, the short side (β) and hypotenuse (δ) of a normalised right triangle with a long side of 1. The first extant column (δ^2) is followed by expanded values b and d and finally the row number.

The 'shape of the triangles varies rather regularly ….'[141] This regularity can be improved significantly.

It is suggested that the operative part was the normalised triangle, with the expanded integer values only required to calibrate an instrument, consisting of an upright of length 1 and a horizontal bar of the same length. The horizontal bar could be moved length-wise, so that the vertical would divide it into two portions with lengths β and 1-β. There would then be two right-angled triangles, sharing a common long side of 1, with sides β, 1, δ, as defined in the tablet, and 1-β, 1, $\sqrt{(2-2\beta+\beta^2)}$ or $\sqrt{(1-2\beta+\delta^2)}$, in the ancillary triangle, which could both be scaled, as required.

Scaling makes no difference to the angles in the two triangles. In the defined triangles the angles change by ca. 0.94° per row, but in the ancillary triangle it is about 1.5°, an attractive 1/60th of a quadrant.

Figures 10 and 11 plot the relationships between the angles and the short sides or the diagonals of the two triangles, several of which are closely linear up to about row 15. The ratios depend on the scaling of the triangles, which is assumed to be by a factor of 11, which is appropriate for the latitude of Babylon (32.5°). There the tangent of the celestial equator (57.5°) is 11/7. The smaller angles in the defined triangles for rows 14 and 15 are 33.3° and 31.9°, with the latter being most appropriate for latitude 31.9°. It has been argued that the tablet was from Larsa on latitude 31.2°, a little south of Babylon.

The ratios of degrees per unit of length are very close to 5° for:

139 Finkel I. The Ark before Noah, Hodder & Stoughton, 2014, p 108. No units are actually mentioned.
140 Brittan J.P. et al, Plimpton 322: a review and a different perspective, Arch. Hist. Exact Sci. (2011) 65 pp 519/566.
141 Neugebauer)., The Exact Sciences in Antiquity, Dover, New York, 1969, p.38.

The short sides of both triangles and the smaller angles in the ancillary triangles
The diagonals and the interior angles of the defined triangles.

The diagonals of the defined triangles and the angles of the ancillary triangles have a ratio of about 8°

It would be simple to change the two ratios from 5° and 8° by increasing the length of the long side from 11 to 22 or 44 respectively to give 2.5° and 2° per unit, the two ancient norms. The alternative is simply to reduce the size of the unit of measurement.

If the small angle in the ancillary triangle corresponds to the zenith distance of a star that transits overhead, the ratio of the east/west co-ordinate of the observer's eye is 6° (time to transit) per unit (see last three columns in Table 24 and figure 12). Such stars were known as *zigpu* stars at the time of mul-Apin, ca. 1000 BC.[142]

Plimpton 322 looks like a multi-purpose tool for astronomers, but this conclusion has been overtaken by the recognition of its more specific use, namely to set out a rectangular building with a diagonal of specific length and orientation (see section10 below especially pages 57/58 and figure 21).

[142] Hunger H. & Pingree D., MUL.APIN, An Astronomical Compendium in Cuneiform, Archiv fur Orientforschung, Beiheft 24, Horn, Austria, 1989 pp 141-144. Walker C. (editor), op.cit. 1996, p.48 refers to 'A number of Late Assyrian observations and of Late Babylonian eclipse reports are timed in relation to the meridian passage of one of a group of stars known as *zigpu* stars.

Table 24. Plimpton 322- values for rows 1 to 15, after scaling the common long side to 11 units.

	1. Defined Triangle			2. Ancillary Triangle			Stars with Declination 32.5° On latitude 32.5°		
Row	β	δ	smaller angle	11-β	diagonal	smaller angle zenith distance	Time to transit	Position Observer's eye	
	units	units	degrees	units	units	degrees	degrees	units E/W	units N/S
1	10.91	15.49	44.76	0.09	11.00	0.48	0.57	-0.09	0.00
2	10.72	15.36	44.25	0.28	11.00	1.48	1.75	-0.28	0.00
3	10.54	15.24	43.79	0.46	11.01	2.37	2.81	-0.46	-0.01
4	10.36	15.11	43.27	0.64	11.02	3.35	3.97	-0.64	-0.01
5	9.93	14.82	42.08	1.07	11.05	5.55	6.58	-1.07	-0.03
6	9.75	14.70	41.54	1.25	11.07	6.50	7.71	-1.25	-0.05
7	9.33	14.43	40.32	1.67	11.13	8.61	10.21	-1.66	-0.08
8	9.16	14.31	39.77	1.84	11.15	9.52	11.29	-1.84	-0.10
9	8.82	14.10	38.72	2.18	11.21	11.22	13.31	-2.18	-0.14
10	8.42	13.85	37.44	2.58	11.30	13.19	15.65	-2.57	-0.19
11	8.25	13.75	36.87	2.75	11.34	14.04	16.66	-2.74	-0.22
12	7.70	13.42	34.98	3.30	11.49	16.72	19.85	-3.29	-0.31
13	7.38	13.25	33.86	3.62	11.58	18.22	21.64	-3.60	-0.37
14	7.22	13.16	33.26	3.78	11.63	18.99	22.56	-3.76	-0.40
15	6.84	12.96	31.89	4.16	11.76	20.70	24.60	-4.13	-0.48
Overall range	4.07	2.53	12.87	4.07	0.76	20.22	24.03	4.04	0.48
Ratio °/β			3.16			**4.97**			
Ratio °/δ			**5.09**			26.61			
Ratio Ancillary Angle °/ δ			**7.99**			16.93			
Ratio angle °/row			0.92			**1.48**			
Ratio Altitude Per E/W unit °/unit							**5.95**		

6. Shadow Lengths - Egypt and Mesopotamia.

Egypt

There are simple portable L-shaped sundials from Egypt dating to the middle of the second millennium B.C.[143] They consist of a short, flat-topped, upright and a long flat horizontal bar to receive the shadow. The gnomon in surviving examples is very short, but some have vertical holes indicating that the height could be raised by the addition of another block. A late hieroglyph even indicates that one gnomon was like a short ladder with 3 different levels.[144] We know that the marks on the horizontal bar are placed, in an

[143] Symons S, Ancient Egyptian Astronomy, PhD Thesis, University of Leicester, 1999, pp 127/151. On pp 127/9 she examines one (E1) from the reign of Tuthmosis III, where the distances between adjacent individual hour marks are 1 – 2 – 3 – 4 – 5 with the marks 1,3,6,10, & 15 units from the gnomon.

[144] Symons S., op.cit. Figure 19c.

arithmetical sequence, at 1,3,6,10 and 15 units from the gnomon. The next two values, in this sequence, would 21 and 28 units, with the latter equally the number of digits in the Royal cubit.[145]

Symons has convincingly argued that those sundials, fitted with a plumb-line to ensure the long bar was horizontal, were designed to be handheld and rotated to point towards the sun.[146] Certainly they could be used in this way, but perhaps more for measuring altitudes rather than estimating time. Comparing the distances, plus or minus 0.5 unit, in the arithmetical series with gnomon height, a gnomon of about 5.5 units would permit good altitude estimates for: 10°, 15°, 20°, 30°, 45°, 60° and 75° (Figure 13).

To measure the same degree values, not of altitude but of time from the rising of the sun, we can calculate the corresponding altitude of the sun at the equinoxes as being: 9.0, 13.5, 17.9, 26.7, 39.5, 51.1 and 60.2. A gnomon of about 5 would give reasonable estimates of time after rising for the first four hours or so. For the remaining hours a shorter gnomon would be required.

We know from the pyramid complex of Pepi II (Figure 9a) that they were particularly focused on the equator, or a little below it. The equinoctial shadow is aligned with the northern edge of the building around the open court. Its eastern edge is about 260 cubits from the centre of the pyramid, equating to 2.6 times the height at ground level. On the roof, if 13 cubits high, the ratio would be 3.0, corresponding exactly to the second mark in the arithmetical scheme. Consequently we can think of the horizontal bar of the sundial as being like that roof, only relatively much longer. [147]

No plumb line is shown in the Osireion drawing and it is suggested that for the estimation of time, throughout the year, the dial was placed due east/west with the face of the horizontal bar flush with the ground.[148] The marks on it could then be extrapolated by eye to the solstice positions or the dial could be rotated about the long arm until the shadow fell on it (table 25).

Table 25. Latitude 26°, Obliquity of Ecliptic 23.83°, 5 unit gnomon, no allowance for refraction, horizontal bar fixed due east/west and flush with the ground..

	Calculated E/W distance	Arithmetical Scheme						
Hour			Difference	Equinoxes	Summer Solstice		Winter Solstice	
	Units	Units	Units	Hours from rising	Hours from rising	Seasonal	Hours from rising	Seasonal
1	20.8	15	-5.8	1.36	**1.41**	1.24	**1.30**	1.51
2	9.6	10	+0.4	1.94	**2.05**	1.80	**1.83**	2.13
3	5.6	6	+0.4	2.86	3.08	**2.71**	2.63	**3.05**
4	3.2	3	-0.2	4.11	4.55	**4.00**	3.67	**4.26**
5	1.5	1	-0.5	5.32	6.00	**5.28**	4.64	**5.38**

The east/west components of the shadows of a 5 unit gnomon, on a latitude of 26°, would be within 0.5 units of four, out of the first five, positions in the arithmetical scheme at

[145] With a one digit gnomon, the altitudes of the shadows corresponding to the first seven positions in the series, would be 45°, 18°, 9°, 6°, 4°, 2.7° & 2.0°. The last, corresponding to one Egyptian Royal cubit of 28 digits, matches one of the two ancient norms in Mesopotamia, with 1 cubit representing 2°.
[146] Symons S., op.cit. p. 143.
[147] The pyramid at Meidum, from ca. 2600 BC has a small chapel on the east and a long causeway, running due east, albeit not horizontally.
[148] Symons S., op.cit. Figure 17, p.131.

hourly intervals (Table 25). The prime reason for the single discrepancy can be attributed to the arithmetical scheme itself, which could easily have had one more position at 21 units from the gnomon, near the end of the bar, for the first hour. The mark at 15 units would indicate 1⅓ hours, not 1, from rising.

The data is broadly consistent with a gnomon of five units on a latitude close to 26° (Figure 13).[149] The dial was evidently intended to indicate seasonal hours, but at the solstices for the first two hours, the times are closer in equinoctial hours. The dial would not show either equinoctial or seasonal hours consistently throughout the year, but was presumably good enough for everyday use.

Once they had recognised that the sun's rays rotated about the top of a gnomon, they could have studied it graphically, just as we can today, albeit with greater ease and precision now. This would explain why refraction seems to have played little or no role. We have already seen above that they were measuring time in units of either 10° or 15° in the pyramid era.

Mesopotamia

Much has been written about the Shadow Length Table in Mul-Apin, but there is one aspect which has still not been resolved.[150] For the equinoxes, no shadow lengths greater than 3 are included, indicating there was an alternative method, other than simply the shadow length, to determine those positions. It was suggested above that in Egypt they extrapolated from the equinoctial positions to those for the solstices. In Mesopotamia they may well have interpolated from the solstices to the equinoxes, graphically by the intersections of the equinoctial shadow path with the straight lines between the points for the two solstices (Table 26 & Figure 15).

Furthest from the gnomon these straight lines mark equal time from rising and lie almost due north/south. Nearer to the gnomon the difference in time from rising, for the two solstices, diverges and the lines deviate further from due north/south. For the first hour or so the table would give quite good estimates of the equinoctial time after rising, but less good thereafter.

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[149] The 26th parallel has interesting properties. Firstly the equinoctial shadow at an altitude of 26.7° is 60° from transit. Secondly, on 26.56°, the equinoctial shadow is exactly half the height of the gnomon from due west/east and the seked (inverse tangent) of the pole is precisely 2. Thirdly, on a latitude of 26.95° and an obliquity of 23.83°, the sun would rise 26.95° either side of due east at the solstices. At a radius of 10 from a gnomon of unit height the north/south distance between the shadows at the solstices would be 17.9 units.[?] With a conventional 180 days between the solstices, each unit would correspond to a decan of 10 days, on average. It is therefore not too surprising that several coffin lid star tables came from Asyut, on latitude 27.2°. (see Symons S., A Star's Year in Calendars and Years (edited by Steele J.M.), Oxbow Oxford 2007), p 2,Table 1.

[150] Hunger H. & Pingree D., MUL.APIN, An Astronomical Compendium in Cuneiform, Berger, Horn, Austria, 1989, pp 153/4.

Table 26. Mul-Apin Shadow Length Table, Latitude 32.5°, Obliquity 23.83°, no allowance for refraction. Indicated times (without brackets) as given in tablet.

Shadow Length	Equinoxes				Summer Solstice			Winter Solstice		
	Shadow length	Time Ind.	Time Calc.	Diff	Time Ind.	Time Calc.	Diff.	Time Ind.	Time Calc.	Diff.
cubits	cubits	degrees	degrees	degrees	degrees	degrees	degrees	degrees	degrees	degrees
10	(8.8)	(7.5)	7.7	**(-0.2)**	6.0	7.6	**-1.6**	9.0	7.9	**+1.1**
9	(7.9)	(9.5)	8.6	**(+0.7)**	6.7	8.4	**-1.7**	10.0	8.8	**+1.2**
8	(7.0)	(10.7)	9.7	**(+1.0)**	7.5	9.4	**-1.9**	11.2	9.9	**+1.3**
7[151]	(6.1)	(12.3)	11.0	**(+1.3)**	8.6	10.7	**-2.1**	12.9	11.3	**+1.6**
6	(5.2)	(14.4)	12.9	**(+1.5)**	10.0	12.4	**-2.4**	15.0	13.3	**+1.7**
5	(4.3)	(17.4	15.4	**(+2.0)**	12.0	14.8	**-2.8**	18.0	16.0	**+2.0**
4	(3.5)	(21.4)	19.0	**(+2.4)**	15.0	18.2	**-3.2**	22.5	19.8	**+2.7**
3	3	25.0	22.0	**+3.0**	20.0	23.7	**-3.7**	30.0	27.4	**+2.6**
2	2	37.5	32.0	**+5.5**	30.0	33.7	**-3.7**	45.0	43.1	**+1.9**
1	1	75	57.0	**+18.0**	60	55.8	**+4.2**	90	73.7	**+16.3**

There is no doubt that the mul-Apin table referred to equinoctial time after sunrise, but there remains the problem with the one cubit length for the winter solstice. For the summer solstice and the equinoxes the length of shadow, when respectively 60° and 75° from rising, would be 0.9 and 0.7 cubits, both close enough to be rounded to 1 cubit. At the winter solstice the shortest shadow is 1.57 cubits, on the meridian, but it is only 74° from rising and therefore far from the 90° of the constant. It is reasonable to consider that it 'was presumably added for reasons of symmetry and to show the value of the constant for that solstice' or the measurements were a little further south.[152]

Hunger and Pingree claimed that 'we must regard the table as based on mathematical manipulation rather than on observation'.[153] Clearly the table incorporates reciprocal relationships, but they must also have had a deep practical understanding of the underlying phenomena (Table 26). The values in the table are after they were forced into the straight jacket of the formulae and so it is likely their underlying data was much more precise. For the equinoxes the fit is not close, presumably because of the 'desire to fix the constant (75) midway between those for the solstices (60 and 90)'.[154]

From Table 26, for a shadow length of 2 cubits at the solstices, the product of the shadow length and the calculated time after sunrise is 67 and 86, compared with the scheme constants of 60 (summer solstice) and 90 (winter solstice).. Figure 16 shows the linear relationship between time and the inverse shadow length and the solstices and equinoxes. For the solstices the linear trendlines indicate ratios of 96 and 56 and also rising H.A. of 256° and 286°, which correspond to declinations of 20.8° and -23.4° and rising azimuths of 65° and 118°. The good fit of the latter, ignoring the 1 cubit value, suggests that the scheme was based primarily on the winter solstice with a constant of 90 and that the 60 and 75 for the summer solstice and equinoxes were derived therefrom.

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151 The table shows no values for this shadow length, because of the difficulty of dividing by 7 in the sexagesimal system, but it is included here for completeness.

152 Bremner, R.W., The Shadow Length Table in Mul.Apin, in Die Rolle der Astronomie, Graz, 1993, p.370. See also Steele J., Shadow-Length Schemes in Babylonian Astronomy, Academia, 2012?, p.11: 'This entry in the scheme is therefore an artefact of the underlying mathematical rule and is, presumably, included in the text either simply for the sake of completeness or perhaps because it is the value of the constant c for that month and so is useful in calculation.' A little south of Babylon the shadow would be under 1.5 cubits, which could be rounded to 1.

153 Hunger H & Pingree D, Astral Sciences in Mesopotamia, Brill, 1999 p.80.

154 Bremner R.W., op.cit. p.369.

7. The 2:1 and 3:2 ratios for longest to shortest day and the Path of Anu.

People all over the world have used the rising and setting of the sun as markers for annual events such as the standstill positions at the solstices.[155] Those living in what is now northern Iraq were surely no different and would have noted the extreme positions of the sun at the horizon. They would soon have realised that these four points, plus the meridian, divided a circle into six equal segments. Adding in the east/west line of the equinoxes gives segments of 30° and we have noted such bearings at Eridu (Latitude 30.5°) around -5000 (Page 12 above). By c.-3100 they were using a star pictogram with 8 points, so by then they were thinking in segments of 15°.

Figure 17 shows graphically the 2:1 and 3:2 ratios for the longest to shortest days, based respectively on azimuth and equinoctial time, at the horizon. The outer time polygon has sides of 24 cubits for 36° time. Interestingly the angle, between the solstices and the equinox, is 18°, similar to that of the oblique palace wall (17°) and to the divisions between the paths of Anu, Enlil and Ea (see footnote 15 above).

Both estimates (15° declination and 17° from due east) for the boundary of Anu stars would be correct on a latitude of 28°, which suggests that the width of the Anu band was more likely to have been determined in the southern, rather than the northern, part of Mesopotamia. Table 27 shows the situation on a latitude of 30° and demonstrates that Anu's limits were probably based on equinoctial times above the horizon with the width being 36° or one tenth of a day.[156] Figure 17 shows the limits of 7 units from the east/west line for the Anu band on a latitude of 35°

Table 27. Latitude 30°. Obliquity of the Ecliptic 23.9°. No allowance for refraction.

Declination	Rising HA	Time above horizon	Rising Az	Azimuth swept
degrees	degrees	degrees	degrees	degrees
23.9	255	210	62	236
Anu 15	261	198	73	214
0	270	180	90	180
Anu -15	279	162	107	146
-23.9	285	150	118	124
Anu range	18	36	34	68
Solstice range	30	60	56	112
Solstice ratio[157]		1.4		1.9

Each 24 cubit side corresponds to 36°(time), giving a ratio of 1.5° per cubit, which with a double cubit would increase to 3.0°. Such a unit would approximate to the ratios implicit

[155] Thurston H., Early Astronomy, Springer-Verlag, New York, 1994, pp 10/11.
[156] On a latitude of 35°, lines of stars with declinations of ± 15° would rise 18° from due east and their time above the horizon would be 202° and 158°. The tangent of 18° is 1/3, which would have been an attraction.
[157] The ratios of 3:2 and 2:1 on latitude 35° were 2.8:2 and 1.9:1 on latitude 30°.

in HS345, summarised as 51 units from the 'Stars' to Bootes and a further 7 units to Scorpio.[158] This is particularly true if the 'Stars', in this instance, should be identified not as the Pleiades but as the Hyades, at least for the overall distances to SUPA and the Scorpion.

Table 28. Summary of tablet HS245, the Hilprecht Text (R.A. for -1600)

	Exemplary Star	R.A.	Degrees from Hyades	Distance	Ratio
		degrees	degrees	units	degrees/unit
Stars	η Taurus (Pleiades)	8	-11		
	θ Taurus (Hyades)	19	0		
SUPA	α Bootes	172	153	51	3.0
Scorpion	α Scorpio	197	178	58	3.1

Table 29 summarises the evolution of ideas about the ratio of the longest/shortest day. It does not include HS245, which pushes the 3:2 ratio back to the Old Babylonian period..

Table 29, Horizon measurements on Latitude 35°, Obliquity 23.9°, no allowance for refraction

	Azimuth swept	Cubits swept hexagon	Cubits swept Stepped curve	Hourlines Hor. Dial	Text BM17175 + 17284[159]	Text mul.Apin	Text mul.Apin	Text Ivory Prism	Length of Daylight
	Degrees azimuth	Cubits along sides	N/S cubits	Degrees from meridian	none	minas	beru	beru	Degrees time
Approx. date	-5000?	?	<-700		-1800	-1000	-1000	<-610	
S. Solstice	240	96	96	120.7	4	4	3.6	8	216
Equinox	180	72	72	90	3	3	3	6	180
W. Solstice	120	48	48	60.4	2	2	2.4	4	144
Ratio	2:1	2:1	2:1	2:1	2:1	2:1	3:2	2:1	3:2
Ratio degrees per unit	1	2.5	2.5	1	60	60	30	30	36

In 1947 Neugebauer was clearly taken with the idea of the 2:1 ratio for the longest and shortest days being based on the use of a water clock, but by 1975 he was rather more cautious.[160] He refers to 'the assumption that the given weights represent the outflow of water from the bottom of a cylindrical container...'. It was only an assumption and in 1996

[158]Hunger H. & Pingree D., Astral Sciences in Mesopotamia, Brill, Leiden, 1999, p.54
[159]Hunger H. & Pingree D. Mul.A[pin, AfO, Horn, Austria, 1989, p 163.
[160]Neugebauer O., The Water Clock in Babylonian Astronomy, 1947, ISIS 37, pp37/43 and HAMA, 1975, p 708.

Hoyrup drew attention to the problems with the water clock model.[161]In 2000 Michel-Nozieres concluded that 'the water weight data … cannot be taken literally'.[162] In spite of Hoyrup's work, Hunger and Pingree in 1999 stated that '1 mina of water in a water-clock measured a third of an equinoctial night', with no caveats.[163]

In mul-Apin the ratio is associated with minas, normally a measure of weight, equivalent to about 500 gms.[164] From school problems from about -1800 we learn of water flowing from a water-clock. However the existence of water-clocks does not mean that a ratio established over millennia, was immediately discarded.

The study by Michel-Nozieres of the problems inherent in outflow clocks found that under the best conditions, the ratio would approximate to$\sqrt{2}$:1, which is far from 2:1. In fact , expressed as 2.8:2, it is obviously much closer to the 3:2 ratio in time.

The 2:1 ratio appears later (pre -611) on an ivory prism as a ratio of angles, expressed in beru (30°) and us (1°), so this same ratio was, in different texts over more than a millennium, based on unstated units, units of weight and units of angle or time. We also have to bear in mind the use of ninda, normally a unit of length, in mul.Apin. After the summer solstice (II I 11/12) 'the sun … turns and keeps moving towards the South at a rate of 40 NINDA per day' and after the winter solstice (II I 17/18) 'the sun … turns and keeps coming up towards the North at a rate of 40 NINDA per day'.[165] In the same section there is reference to the length of the watch in terms of minas, so we appear to have a mixture of units of weight and length.

If, at the time of mul=Apin and before, they could measure time accurately enough in equinoctial units to confirm the 3:2 ratio, it seems somewhat perverse to use simultaneously a 2:1 ratio of weights, unless the two ratios were never intended to refer to the same phenomenon or were not established at the same latitude.

To resolve this issue perhaps we need to take a different approach. When experimenting with water clocks they might have tried weighing the water dripping into a bowl until the scales tipped.[166] This would justify measuring the quantity of water by weight rather than volume. If they were measuring the time for the sun to traverse a large segment of the horizon they might have noticed that it was like the bow wave of a swimming duck. This would justify the association of weight and ducks, with many standard weights being in the form of a duck.[167] However it would imply that 'mina' in addition to its usual meaning of weight was also a segment of a circle. With 6 minas in a full day, each would correspond to 60°. There is further discussion of minas below (p.57)

The 3:2 ratio is extraordinarily precise for northern Mesopotamia. To understand how this might have been achieved, we assume they were aware how the 'shadows' of stars, with

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[161]Hoyrup J., A note on water-clocks and on the authority of texts (pre-print 1996), AfO, 44-45,
[162]Michel-Nozieres C., Second Millennium Babylonian Water Clocks: a Physical study, Centaurus 2000, Vol.42 pp 180/200.
[163]Hunger H. & Pingree D., Astral Sciences in Mesopotamia, Brill, Leiden, 1999, p.46
[164]Hunger H. & Pingree D, Mul.Apin, An Astronomical Compendium in Cuneiform, AfO, Horn, Austria 1989, pp 163/4 (Appendix). The tablets are dated to the old Babylonian period c.-1800.
[165]Hunger H. & Pingree D., Mul.Apin op.cit pp 72/75.
[166]In Portugal many years ago I saw an old domestic water meter which used such a system. When one bowl filled the flow was diverted to fill the other. Each switch being counted to determine the volume.
[167]A water clock with sinking bowls would also remind them of ducks, with both likely to dive suddenly.

the same R.A., cross the meridian simultaneously and rotate, in a sensibly straight line, around the 'pole' on the ground.[168]

In mul-Apin the King (α Leo) is said to rise with the Bow constellation, identified with the southern part of Canis Major.[169] As it rises just 2 minutes ahead of α Leo, o^2 CMa was chosen as the exemplary star for the Bow[170]. Although not especially bright It is part of 'one of the most luminous pairs in the heavens'.[171] Significantly the declinations of the two exemplary stars are close to those of the sun at the solstices and so their 'shadow' paths could be seen as proxies for the paths of the sun's shadows at those extremes.[172] Successive positions of the two stars, as seen from a latitude of 34.75 (44°east) in -1000 are summarised in table 30 below and their 'shadows' are illustrated in Fig.23.[173]

Table 30. 'Shadow' positions of α Leo and o^2 CMa in -1000, when the obliquity of the ecliptic was c. 23.8°. Observer on Latitude 34.75° and 44° east. Data from SkyMap Lite 2005.

		o^2 CMa R.A. 74.7° Decl. -23.8°			α Leo R.A. 110.0° Decl. 22.8°		
Row	Position	Azimuth	Time to transit	Hr-line angle	Azimuth	Time to transit	Hr-line angle
		degrees	degrees	degrees	degrees	degrees	degrees
1	Both on E. horizon	119.4	72.8	61.5	61.5	107.5	61.1
2	o^2 CMa at transit	180	0	0	101.7	35.3	22.0
3	α Leo at transit	215.0	35.4	22.0	180	0	0
4	o^2 CMa on W horizon	241.0	73.0	61.8	260.2	37.7	23.7

The two stars rise together in the east, but o^2 CMa reaches the meridian first (row 2 in Table 30) when α Leo is 35.3° in time from the meridian.[174] Drawing a line from the 'pole' to the 'shadow of α Leo indicates the hour-line angle for that time difference. It can be mirrored by a similar line on the other side of the meridian so that when α Leo reaches the meridian (row 3) the 'shadow' of o^2 CMa. will lie on it.

[168] We have shown that in Egypt they were seemingly beginning to show an interest in the' pole' at Hierakonpolis
c. -3000 (p. 21 above) and by the time of the pyramids they were familiar with the geometry around the 'pole' (p. 30 above).
[169] Hunger H. and Pingree D., Mul-Apin, Horn, Austria, 1989 p.140, Table 1 (Tablet ii.44).
[170] Based on Sky Map Lite 2005 o^2 Cma rose 2 minutes before α Leo and δ Cma 14 minutes after. The latter would have been the more appropriate if the observer was on the lower latitude of 32.5°.
[171] Dibon-Smith, Star List 2000, John Wiley & Sons, 1992, p.44, note k.
[172] In -1000 the obliquity of the ecliptic was c.23.8°
[173] The latitude of 34.75° is propitious because several hour-line angles are particularly attractive: Hour-line angles of 22.5°, 30°, 45° & 60° equate to times to transit of 36.0°, 45.4°, 60.3 °& 71.8°
[174] At that moment α Cmi (R.A. 75.1°) would also be very close to the meridian and the 'shadows' of δ Vel (R.A. 110.1) & λ Vel (R.A. 109.9°) would be almost perfectly aligned with that of α Leo on the same hour-line angle.

By the time o^2 CMa is setting in the west (row 4) α Leo is 37.7° from the meridian and its hour-line angle would be 23.7°, just outside the previously marked radial line for 22.0°. o^2 CMa moved from one horizon to the other in two equal steps, divided by the meridian, while α Leo moved in three similar steps with the two radial hour-line angles for 35.3° dividing them. We now know that the first two time divisions are c.72.8° and the other three c. 72.2°, 70.6° and 72.2°, but at the time it would have been justifiable to consider they were all sensibly the same. With the longest night paired with the shortest daylight a full day would have 5 similar steps and the ratio of the longest and shortest days would be 3:2 with each step being precisely 72°.

Although we have referred to time measurements a ratio of 3:2 could have been logically deduced solely from the positions of the star 'shadows' in relation to hour-line angles. c.22° either side of the meridian, without any direct measurement of time.

This is not to infer that they were incapable of handling time degrees and in the, albeit later, tablet BM29371 there is a column which explicitly gives half-day lengths as 72° and 108° at the winter and summer solstices.[175] Furthermore there is another column, with no heading but with 'kila' in the first line, which runs from 1 to 1.5, suggesting that possibly 'kila' was equivalent to 72° (time).

[175] Walker C., Astronomy before the telescope, British Museum 1996 p.47 and see also Brown D., Fermor J., and Walker C., The Water Clock in Mesopotamia, Archiv fur Orientforschung, They note the early appearance of the 3:2 ratio in a seventh century B.C. version of mul-apin.

8. Djed Pillar and Time Measurement.

The vertical Djed pillar in Figure 14 vaguely hints that it might be related to the measurement of time using a horizontal sundial. On the other hand Figure 18 shows a modern drawing of the hour-lines for an east-facing vertical sundial with a style aligned to the pole and also a Djed pillar at Abydos (latitude c. 26°) inclined at c. 25° from the vertical and surmounted by twin plumes.[176] The two are remarkably similar. The width of the 'pillar' corresponds to the length of the style and the hour-line positions depend on the height of the style away from the meridian plane. In this type of vertical dial the longest shadows are at mid-day and the shortest at the horizon. The 'pillar', on which the shadows fall, is inclined from vertical at an angle corresponding to the latitude of the site.

The Djed pillar symbol itself dates back to pre-historic times, but this does not imply that it was always associated with the measurement of time.[177] It could be that when this type of sundial was developed, someone noticed that the shadow lines looked like a leaning Djed pillar, whatever that might have been. The ritual of 'raising the djed pillar', is known from the Old Kingdom at Memphis, which suggests the possibility that the association with time was established by say 2500 BC.[178] This date coincides with the growing importance of the east/west line (cf Menkaure's pyramid causeway) and the size of the mortuary chapels and other buildings immediately east of pyramids.

The Djed pillar symbol, and presumably its dialling properties, reached Mesopotamia from Egypt around -1800[179]. From about -500 there is a shadow table (BM29371) with intervals of 5 days, against each of which is written 'One cubit shadow, 1 $^{2}/_{3}$ double-hours day'.[180] This has been interpreted as meaning 'after 1 $^{2}/_{3}$ double-hours of day the shadow of the gnomon has a length of 1 cubit', throughout the year. If 1 $^{2}/_{3}$ double hours equates to 50° (time), then an east-facing vertical gnomon with a style of $^{5}/_{6}$ cubit, would have a shadow of 1 cubit.[181]

9. Ready Reckoner for converting rising azimuth to rising time.

There is an alternative to the concentric polygons in Figure 17. We have already noted the stepped curve for the linear measurement of azimuth, so it is likely they would have sought a similar curve for the measurement of time. On a latitude of 35° the rising sun at the solstices would be 36° (time) apart and approximately +/- 30° from due east[182]. On the stepped curve for azimuth, the sun would be 12 (30/2.5) cubits north or south of due east and for that same N/S distance to suspend 18° (time), the distance along the east/west line

[176]Cousins F.W., Sundials, Redwood Press, Trowbridge, 1972, p.132 and Lurker M., The Gods and Symbols of Ancient Egypt, Thames and Hudson, London 1982, p.47. There is a large ancient Greek sundial with similar curves at the British Museum (ref:1816,0610.186). It is inscribed 'Phaidros, son of Zoilos'.
[177]Shaw I. and Nicholson P., British Museum Dictionary of Ancient Egypt, London, 1997, p.86. On page 304, they mention the possibility that the was sceptre was used as a gnomon and it might be seen as stripped down version of a vertical dial, facing east or west, with the angled head pointing to the pole.
[178]Lurker M., op.cit, p 47.
[179]Black J. & Green A., Gods, Demons and Symbols of Ancient Mesopotamia, British Museum, 1992, p.74.
[180]Britton J. & Walker C., Astronomy and Astrology in Mesopotamia (in Astronomy before the Telescope), British Museum, 1996, p.47. More recently in Steele, J. Shadow-Length Schemes in Babylonian Astronomy, Academia, 2012? pp 30ff there is a different interpretation of the text.
[181]The calculation is Tan 50 x 5/6.
[182]On a latitude of 35° and an obliquity of 23.8°, the sun at the solstices would rise 29.5° north or south of the east/west line and would take 90 +/-18° (time) to reach the meridian. The ratio of the longest to the shortest day would 108/72 or 3/2, an ancient Babylonian norm.

would be 36.9 cubits.[183] Rounding down to 36 cubits and, assuming 2° per cubit, would indicate a time difference of 18° between an equinox and a solstice. It would be a simple matter to increase the dimensions of the stepped curve by 25% and rotate it so that the long axis lay due east/west. With each east/west cubit equalling 2° time, the furthest point would be 45 cubits from the centre, corresponding to 90° of time to the meridian at the equinoxes (Figure 19). The section of the time curve between solstice and equinox is sensibly linear, lying between +/-12, 36 and 0,45 and resembles the hypotenuse of a 3,4,5 triangle, scaled up by a factor of 3.

There would be near linear relationships between declination, rising time and rising azimuth and also the cubit measures of rising time (at 2.0°) and rising azimuth (at 2.5°) It would exploit the linear relationship between rising azimuth and time to the meridian by using both of the two ancient norms for the ratio of degrees per cubit.[184] Table 22 demonstrates how closely the results of such a 'ready reckoner' would match modern calculations.

The proposed time curve does not allow measurements across the east/west line. For any body with positive declination and rising north of that line, it is necessary to add 45 cubits to the equatorial distance. For the summer solstice this means adding 45 and 9 to give 54 cubits. Graphically this is like measuring to a mirror image, shown dashed in Figure 19. At the winter solstice the distance is 36 cubits, a difference of 18 cubits or 36° (time).

If correct, the ready-reckoner must surely represent a high point in the use of linear cubits to represent angles. However it does have a disadvantage: The three months between solstice and equinox are not distributed evenly along the 15 cubit hypotenuse with the divisions between them being at 7.1 and 12.6 cubits from a solstice. This can be remedied by changing the X-axis from cubits to days.

In a schematic year of 360 days, there are 180 days or 180° longitude, between solstices, so the average daily change in time, would be 18/180 cubits equivalent to 0.2 cubits or 12 minutes. Each east/west cubit would equal about 10° longitude. However using longitude (or days), as shown in red in Figure 19, means the loss of the near linear relationship between rising azimuth and rising time, when working solely in cubits.

[183] 12/Tan (18) equals 36.93 cubits.

[184] The ratio between time and azimuth is about 0.6 for latitudes 32.5/38° and 0.5 for 25/30°. The latter would require a pair of stepped curves with ratios of 2° (time) and 3°(azimuth), instead of 2.5°. The assumed solstice positions being 27° from due east and 13.5° (time) from an equinox, both correct about 27.5° (latitude).

Table 31. Columns 1/5 are modern calculations for 35° latitude, 23.8° Obliquity, with no allowance for refraction. Column 6 is azimuth from winter solstice (assumed to be at 60° from the meridian) divided by 2.5. Column 7 is column 6 times 1.5 (0.75 x 2), Column 8 is the difference between modern calculations and the 'ready reckoner'. Column 9 is the daily change and Column 10 is the similar modern calculation.

Long	RA	Decl.	Time from solstice	Rising Azimuth	Rising Azimuth from W.S.	Time from winter solstice	Differen ce	Daily Change	Daily modern calculation
1	2	3	4	5	6	7	8	9	10
degrees	degrees	degrees	degrees	degrees	cubits	degrees	minutes	minutes	minutes
0	0	0	18	90	12	18	0		
1	0.9	.4	17.7	89.5	11.8	17.7	-.1	17.7	16.9
15	13.8	6.0	13.8	82.7	9.1	13.6	-0.7		
16	14.7	6.4	13.5	82.2	8.9	13.3	-0.7	17.0	16.5
30	27.8	11.6	9.7	75.7	6.3	9.4	-1.0		
31	28.8	12.0	9.4	75.3	6.1	9.2	-1.1	15.8	15.7
45	42.5	16.6	6.0	69.6	3.8	5.8	-0.8		
46	43.5	16.9	5.7	69.2	3.7	5.5	-0.8	13.2	13.7
60	57.8	20.5	2.9	64.7	1.9	2.8	-0.1		
61	58.8	20.7	2.7	64.5	1.8	2.7	0	9.6	10.4
75	73.7	22.9	0.8	61.6	0.6	1.0	0.8		
76	74.8	23.1	0.7	61.4	0.6	0.9	0.8	5.1	5.7
85	83.5	23.7	0.1	60.7	0.3	0.4	1.1		
86	84.5	23.7	0.1	60.6	0.2	0.4	1.1	1.8	2.0
90	90.0	23.8	0	60.5	0.2	0.3	1.1		
91	91.1	23.8	0	60.5	0.2	0.3	1.1	-0.2	-0.2

The x-axis covers 90° longitude (or days) and the y axis the rising azimuth in cubits, equalling 2.5°. At 30 day intervals the y axis values (Table 23, col.6) are 12, 6.3, 1.9, 0.2 cubits, implying respectively 0.19, 0.15 and 0.57 cubits/day on average. The three initial values in the Jupiter tablets are 12, 9.5 and 1.5 minutes or 0.2, 0.16 and 0.25 degrees.[185] The match is least satisfactory around 20° before a solstice.

At first sight, the Jupiter values closely match those for the sun in Table 31, particularly if the latter represent 30 day averages, and provide support for the 'ready-reckoner' hypothesis. The overall slope of the curve is dictated by the relative size of the two stepped curves and the closeness to the Jupiter values suggests that they were indeed using 2.5° and 2° per cubit for the sun. However closer inspection shows that firstly, as already

[185]Ossendrijver. M, Ancient Babylonian astronomers calculated Jupiter's position from the area under a time-velocity graph, Science Vol. 351, Issue 6272 pp 482/4, Jan 2016. His figure 2 shows three values beginning at 12 minutes per day, with 9.5 and 1.5 minutes per day 60 and 120 days later. The first equals the average rate of change, between solstices, of 36° in 180 days, mentioned earlier.

noted, one refers to time and the other to linear cubits and secondly the number of days between the extreme values are 120 and 90. Ossendrijver has demonstrated that they were measuring time rather than rising azimuth and we must therefore consider the possibility that they were measuring both and the Jupiter data was of particular interest, at the time, precisely because the numerical values were similar to those of the sun, albeit using different units, over different time spans. Table 32 summarises the data.

Table 32. Jupiter's path for the 120 days before a specific first standstill position, when Jupiter is close to the sun in mid-winter. The basic data is for 12/12/2018 to 11/4/2019, but applied to a latitude of 35°, with 23.8° for the obliquity of the ecliptic. The data for the sun is from Table 31 (col.6).

Jupiter (R.A.)				Sun (rising time)	
Days before standstill	R.A.	Change R.A. to next day	Jupiter tablet values	Days before Winter Solstice	Av. Change over 30 days, in N/S cubits
	degrees	minutes	?	days	1/60th cubit
120	245.9	13.8	12.0	90/60	11.4
90	252.6	12.6			
60	258.4	10.2	9.5	60/30	8.8
30	262.4	5.4			
0	263.9	0	1.5	30/0	3.4

Furthermore on this small part of its orbit Jupiter moves 18° R.A. in 120 days while the sun's rising time changes 18° in 90 days.

10. Orientation of Temples etc in Upper Egypt.

Temples and Perimeter Walls at Karnak

Firstly we need to appreciate how the Egyptians handled slopes or angles. They used seqeds which we would now describe as cotangents or inverted tangents.[186] This is similar to how a modern roofer draws the desired angles, using a 'square', before cutting his roof timbers or how hill slopes used to be described as, for example, 1 in 10.[187]

Shaltout and Belmonte give the azimuth of three 18th Dynasty temples in the Amon precinct as 116.75° and that of Montu as 27°, which can be considered as 26.75° from dueeast and 27° from north.[188] These orientations are similar to the main alignments at

[186]Gillings R.J. Mathematics in the Time of the Pharaohs, Dover Publications, New York (1982) p185
[187]Lehner M., The Complete Pyramids, Thames and Hudson, London (1997) p210 illustrates an ancient ' Egyptian square' and Draper J.T. The Steel Square applied to Roof Construction, The Technical Press Ltd., London (1930) gives the tangent ratios for 116 different angles between 29.92° and 78.68°.
[188]Shaltout M. and Belmonte J.A., On the Orientation of Ancient Egyptian Temples: (I) Upper Egypt and Lower Nubia, Journal for the History of Astronomy, xxxvi (2005) 273-298, Table 1. They list 23 temples, but some later ones have been omitted to focus on a shorter time frame. They note two temples of the 11th Dynasty at Thoth Hill and Deir Bahari, with azimuths of 117° and 118.25° respectively, but by the 18th Dynasty a third of all the temples had orientations to the cardinal directions between 25.5°/28.5°. The frequency falls thereafter.

Nabta Playa, three millennia earlier, with the cotangents of the angles being close to 2 (26.6°).[189]

The orientation of the Mut temple is given as 18° with a cotangent close to 3 (18.4°), which is similar to the 18.5° for a line of post holes at Hierakonpolis, two millennia earlier (see above p.21).

There are several reasons why ancient Egyptians might have found these two angles attractive:
a. simple cotangents ratios: 2 for 26.565° and 3 for 18.435°,
b. their sum is exactly 45°, or half a quadrant, with a cotangent of 1,
c. the difference between them is 8.130° with a cotangent of 7,
d. the two angles, 26.565° and 18.435°, are half those in the simplest Pythagorean triangle, with sides in the ratio 3,4 & 5. Those angles, 53.13° and 36.87°. have cotangents 3/4 and 4/3,
e. The larger of the two angles is close to the rising angle of the mid-winter sun, which reported recently corresponds to c. 26.8° from due east.[190] With the small change in the obliquity of the ecliptic we can conclude that at the time the rising angle would have been closer to 27.3°,[191]
f. At Karnak the celestial pole had an altitude of 25.72° and the mid winter sun would have transited at an altitude of about 40.4°, with a tangent ratio close to 6/7 (40.6°), and in mid-summer at 88.2°, close to the zenith.
g The main temple axis was perpendicular to the Nile.

The first four reasons are mathematically exact whereas the last three are dependent on geographic latitude. However cotangent 2 was also employed some distance north and south of Karnak, including Timna (latitude 29.77°) and Abu Simbel (latitude 22.34°).[192] There the rising azimuth of the mid-winter sun would differ from that at Karnak by +1.1° at Timna and -0.75° at Abu Simbel. So, even though making cotangent 2 even more attractive, the prime justification for the precise layout at Karnak was the geometry. At Karnak an azimuth of 116.75° was found to correspond to the rising of a celestial object with a declination of -24.2° (see Table 32).

Most of the temples listed by Saltout and Belmonte fall into one of two groups with cotangents of 2 or 3, corresponding to azimuth angles, from a cardinal direction, of 25.5°/28.0° and 17.5°/19.5°, but two imply cotangents of 1.5 and 2.5 (see below for 2.4 for the southern part of the Amon perimeter wall).[193]

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[189]Malville J.M.,Schild R., Wendorf F. & Brenmer (sic) R., Astronomy of Nabta Playa, in Holbrook H. et al, African Cultural Astronomy, Springer (2008), 131-143, 137.
[190]Furlong D., Egyptian Temple Orientation (part 2), www.davidfurlong.co.uk/egyptarticle gives the main temple alignment of 296°53' or 296.88°. Averaging that with he 116.75°, in the opposite direction, from Table 32 gives 116.82°.
[191]Bretagnon P., Rocher P{. & Simon J.L. (1997) Theory of the Rotation of the Earth, Astronomy & Astrophysics 319 (1997) p305-317.
[192]For the Abu Simbel details see footnote 180 above and for Timna see Belmonte J.A. et al , On the Orientation of Ancient Egyptian Temples (5), Journal for the History of Astronomy xli (2010), 1-29, Table 1.
[193]The angular difference between cotangents 2.4 and 2.5 is 0.82°.

Table 33. Details of sixteen, mainly 18th Dynasty, Temples at Karnak (Lat. 25.72°).The basic data is from Shaltout and Belmonte's Table 1 (see footnote 180), but omitting six later temples and reordered with descending cotangents.

Temple	Dynasty	Azimuth	Azimuth from N, E, S or W.	Horizon Altitude	Declination	Cotangent (Az)
		degrees	degrees	degrees	degrees	ratio
Kamutef	20	287.5	17.5	3.5	17.2	3.17
Boat Station	18	107.5	17.5	0	-16	3.17
Mut	18	18	18	2.0	60.4	3.08
Khonsuupakherd	18-21	289	19	3.5	18.5	2.90
Rameses III	20	19.5	19.5	2.0	59.5	2.82
Amenhotep II	**18**	**291.5**	**21.5**	**3.5**	**20.8**	**2.54**
Maat	18	205.5	25.5	4.0	-51.5	2.10
Sethy II	19	206	26	0	-54.5	2.05
Ramesses III	20	26.5	26.5	0	53.3	2.00
Amon (main)	12/19	296.75	26.75	3.5	25.4	1.98
Sun High Place	18	116.75	26.75	0	-24.2	1.98
Hatshepsut	18	116.75	26.75	0	-24.2	1.98
Re- Horakhty	19	116.75	26.75	0	-24.2	1.98
Montu	18	27	27	0	53.0	1.96
Raet-tawy	18	28	28	0	52.3	1.88
Ptah	**18**	**304.5**	**34.5**	**3.0**	**32.0**	**1.46**

The perimeter wall is dated to the 30th Dynasty, more than a millennium after many of the temples in Table 33. The western and eastern sections are perpendicular to the main axis of the temple complex and consequently have the same cotangent of 2 for their orientation to the meridian.

In Figure 20 right-angled triangles were drawn, using cotangent 2, with the two parallel sides of the wall as hypotenuses. Their short sides were then extended to produce right-triangles on the northern and southern sides. with cotangents 1.5 and 2.4 and an inner rectangle with sides oriented to the cardinal points. The meridian through its centre cuts the main axis near the western face of the 4th pylon, constructed by Thutmose I.[194]

Three published plans of the layout at Karnak were examined to see how closely they matched the theoretical values, particularly of the four corner angles, and that in Shaw and Nicholson, was the closest with only a small difference in the southern corners.[195] It is the basis for the simplified drawing in Figure 20, which is drawn with the perimeter corner angles in accordance with the theory. Its western and eastern sections are parallel and with the main temple axis perpendicular to them both. That axis is 26.565° from due east or 116.565° from north. Shaltout & Belmonte recorded four temples oriented 116.75/296.75° and with Furlong's figure of 116.88 ° gives an average of 116.8°.

194 Wilkinson R.H., The Complete Temples of Ancient Egypt, Thames & Hudson, London (2000) p.158
195 Shaw I.,& Nicholson P., British Museum Dictionary of Ancient Egypt, London, 1997, p.147 Plan of Karnak.

The two obtuse angles in the NE and SE exceed 90° by 7.125° and 3.945° with cotangents of 8 and 14.5 respectively.

For the Mut complex a triangle with cotangent 3 was visualised with its small angle in the middle of the third court. It aligns with the causeway and axis of the Mut temple and also the western side on the avenue leading to Luxor.[196] The perimeter wall there is not so carefully laid out. The eastern section is parallel to the similar wall of the Amun precinct, but the western wall is misaligned by over 1°, which makes it more difficult to determine the intended orientations, but it is possible the layout had the same two triangles on the west and east with, on the north, a triangle incorporating a cotangent of 3, while on the south the triangle may have incorporated an angle with a cotangent of c. 1.08, possibly 13/12 (42.71°).

The azimuth angles have simple cotangent ratios. Consequently they do not necessarily imply direct astronomical associations, but they may well do so indirectly. When an interesting azimuth was observed, it had seemingly first to be associated with the nearest angle with a known cotangent ratio before any monuments were actually built. This was presumably because accurate construction without such known ratios would have been more difficult.

It has been shown above that at Hierakonpolis they had already recognised three Pythagorean triangles with ratios of the sides: 3, 4 & 5, 5 12 &13, 9, 40 & 41, but at Karnak they had progressed beyond the few such simple ratios. By the 12 or 13th Dynasties, they were calculating square roots, but it is not essential to assume that they were doing so at Karnak.[197] Within say a 3,4,5 triangle to provide an accurate right angle, they could have experimented graphically with different lengths of side to find hypotenuses with preferably integer digit values or at least simple fractions of a digit. With the length of the hypotenuse thus determined, they could prepare a loop of rope with a length equal to the sum of the three sides and with marks for the three corners. Stretching the loop around two posts, marking the ends of one side and aligned with a chosen direction, would determine the third corner. Reversing the rope to the other side would give the fourth corner, thus ensuring both the proportions and orientation were correct. It is reminiscent of the "stretching the cord" ceremony for laying out a ground plan, known from the earliest dynasties.[198]

The main elements of the temple layouts must have been established by the time of the 4th pylon or at least the erection of the nearby obelisks by Thutmose I and his daughter, Hatshepsut .

When much later the perimeter wall was built existing structures were demolished to allow the construction of the eastern section, which we have considered as being one of the two prime axes at right angles to the main temple alignment. Certain features marked two meridian lines: one from the north-western corner to where the south-eastern corner of the Fourth Court met the southern perimeter wall and the other from the south-eastern corner to the gateway in the middle of the northern section near the temple of Ptah. The inner core of that temple was constructed by Thutmose III, implying its foundations were a little earlier.

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[196]The latter is not shown in Fig.20, but appears in a satellite image in Egyptian Archaeology 61, p.11.
[197]Gillings R.J. Mathematics in the time of the Pharaohs, Dover Publications, New York (1982)
[198]Clagett M., Ancient Egyptian Science, American Philosophical Socirty Vol.3 (1999) p.5

Within the temple structures themselves the best indicators of the meridian would be the diagonals across the Great Hypostyle Hall and the Festival Hall of Thutmose III, as indicated in Figure 20.

The 10th pylon, eventually incorporated in the southern perimeter wall, was built in the time of Horemheb, at the end of the 18th Dynasty. [199] The alignment of the northern perimeter wall is the same at that of the Ptah temple, built earlier in the 18th Dynasty. The main axis was clearly defined by the time of Thutmose I, but may have originated earlier. Therefore although the perimeter wall itself was only built in the 30th Dynasty the alignments of each of its four sides are the same as those of older structures.

In -1500 the obliquity of the ecliptic was 23.87°, 0.4° greater than at present, which implies a higher rising azimuth of c.0.5° for the mid-winter sun. The main axis of the Karnak temples lies on a bearing of 116.8° and, according to Furlong's photograph, is currently aligned with the solstitial sun. The estimated azimuth of the midwinter sun, at the time, must have been nearer 117.3°. The difference of 0.7° between that value and 116.6°, might be due to an error in their estimate of the true meridian or was the closest they could get with a familiar cotangent.

Conclusions.

The layout of the Amon precinct at Karnak was based on right-angled triangles with simple cotangents. Although the main axis was in close alignment to the rising of the mid-winter sun, that alone was an insufficient justification for its orientation. There was also a prerequisite to identify a right-angled triangle, with a known cotangent ratio, which, as closely as possible, aligned with the rising of the sun or other body. Only then were the temples actually laid out. This would have helped ensure the buildings were rectangular, with the correct proportions and desired orientation.

The Egyptians were clearly masters of geometry, as Strabo claimed. The layout of the Karnak temples and walls was surely the acme of the system of using cotangents , or seqeds, to describe angles. After millennia the system was coming to the end of its useful life and giving way to less simple cotangents and Pythagorean triangles and eventually to the direct measurement of angles, culminating in there being 360° in a circle. Before then Posidonius referred to 'parts', 1/48th of a circle or 7.5°, and interestingly the cotangent of 7.5° is 7.6, so providing a possible link to the older system.[200]

The orientation of temples in Upper Egypt (including Karnak), prior to the Ptolemaic period.

This study is based on the azimuths of 106 structures recorded by Belmonte et al, with their azimuths adjusted to angles, measured either clockwise or anti-clockwise from a cardinal direction.[201] Consequently their absolute overall range is from zero to 45°, the

[199]Wilkinson op.cit. p.161.
[200]Neugebaquer O., A History of Ancient Mathematical Astronomy, Part 2, Springer-Verlag, New York 1975, p.671.
[201]Shaltout et al in (footnote 180) listed 133 structures, which was supplemented by a further 34, south of the 29th parallel, from Belmonte J.A. et al, On the Orientation of Ancient Egyptian Temples (4), Journal for the History of Astronomy (2008) p.185, Table 1. From that total of 167, 61 from the Ptolemaic and Roman periods were removed to leave 106.

maximum smaller angle in a right-angled triangle. Their dates run from the Archaic period to the 30th Dynasty.[202]

Overall there were 53 different absolute bearings.[203] The most frequent were 26.75° (7 at Karnak and Abydos, all before the end of the 19th Dynasty) and 42.5° (8 at M.Habu, Abydos, Beit el wali, Karnak, Luxor and Qurna all from the 19th Dynasty or later). Not only were these the most common, there were also many others close in bearing : 20 between 25.5°/27° and 21 between 41.5°/43.5°. Of our total of 106, 41 were in these two narrow bands.

Absolute azimuths, 40° or over, were 20% of the total in the 18th Dynasty, 36% in the 19th and 50% in the 20/29 Dynasties, but by the 30th there were none. The earliest recorded use of angles between 41° and 45° can be seen in Table 34.

Table 34. The largest different absolute angles employed in the layout of 28 temples, out of a total of 106. The two from Kom el Ahmar, within square brackets, were considered as being too early.

Short side length (angle)	Belmonte et al angle (no. of examples)	Reference publication & table line number	Location	Reign	Approx date
52 (40.9)	41 (2)	1/44, 1/75	Luxor	Seti I	-1280
52.5 (41.2)	41.25 (1)	4/24	Abydos	Dyn 4	-2550
53 (41.5)	41.5 (3)	1/60, 1/101, [4/41]	Qurna	Ramesses II	-1270
54 (42.0)[204]	42 (4)	1/2, 1/67, 1/104, 1/128	M.Habu	Ay-Horenheb	-1320
55 (42.5)	42.5 (8)	1/28. 1/43, 1/63, 1/70-72, 1/120, 4/31	Luxor Beit el Wali	Ramesses II	-1270
56 (43.0)	43 (2)	1/62, 1/93	Qurna El Qab	Thutmose IV Amenhotep III	-1400 -1390
57 (43.5)	43.5 (3)	1/3, 1/61, 4/18	Abydos Qurna	Ramesses II	-1270
58 (44.0)	44 (1)	1/1	Abydos	Dyn 2	--2700
59 (44.5)	44.5 (3)	1/73, 4/32, [4/42]	Malqata Abydos	Amon Khentamentyu	-1270 -1150?
60 (45.0)	45 (1)	1/59	Qurna	Amenhotep II	-1420

[202]There were three from the Archaic (1) and Pre-dynastic (2) periods and 20 later than the 19th dynasty. One was undated and seven had a range of dates.
[203]Applying the same definition of 'absolute' bearings to the slopes of the generally earlier pyramids , eight appear to be similar to Belmonte et al degrees (cotangent ratio): 34° (28/19), 35.5° (7/5), 37° (4/3), 37.5° (56/43), 38° (14/11), 40° (56/47) 40.5° (7/6), 41.25° (8/7), with the last featuring in Tables 34, 35 & 36 and 35.5° in Table 35. In all but the last the numerator of the cotangent ratio is a simple fraction of 56, not 60.
[204]The third and fourth rows with short sides of 53 & 54, with a long side of 60, are close to the fifth and sixth rows in the Plimpton 322 tablet with angles 42.1° and 41.5°.

By the time of Ramesses II, the triangular set-up with a long side of 60 units was clearly in use for angles between 41° and 45°, where each unit along the short side corresponded to 0.5°. There are earlier examples, but they do not necessarily require a long side of 60 units. The 58/60 ratio from the 2nd Dynasty may simply be an imprecise 45°. The 45° angle itself may just be a convenient half of a quadrant and the others of 52.5/60 and 56/60 may simply be 'nice' cotangents of 8/7 and 15/14.

For absolute angles below 40° there seems to be no evidence that they tried to maintain a similar linear relationship between the length of the short side and the angle, by progressively increasing the length of the long side from say 60 to 72, 78 and 84 units.

Belmonte et al recorded their azimuths to the nearest 0.25° and their measurements include 44 with integer values with a further 4, 50 and 8 including fractions of ¼, ½ and ¾ respectively.[205] In this unusual distribution, the overwhelming majority were either whole or half integer degrees, presumably reflecting the original layout precision.

The 12 examples with the smallest subdivisions are shown in Table 35 and the adjusted angles can be associated with Pythagorean triangles, although the two from the 4th and 11th dynasties are actually closer to simple cotangents.

Table 35. Twelve temples with bearings incorporating ¼ and ¾ fractions of a degree.

Location	Dynasty	Azimuth/ Abs. Az	Cotangent (Angle)	Pythagorean Triangle (Angle)	Decl
		degrees	ratio (degrees)	side lengths/degrees	degrees
Abydos	4	48.75/41.25	**8/7 (41.19)**[206]	48/55/73 (41.11)	36.0
Deir Bahari Elephantine	11 18	118.25/28.25	**13/7 (28.3)**	8/15/17 (28.07)	-25.5 -24.8
Karnak	29	286.25/16.25	17/5 (16.39)	**7/24/25 (16.26)**	15.25
Abydos (3) Karnak (2) Karnak Karnak	18 18 19 12-19	26.75 116.75/26.75 116.75/26.75 296.75/26.75	2.0 (26.57)	**105/208/233 (26.78)**	52.75 -24.2 -24.2 25.4
Abydos	18	31.75	8/5 (32.01)	**28/45/53 (31.89)**	49.25

Half the azimuths are roughly directed towards the outer limits of the sun's annual movement along the horizon, although the overall declination range of 49.6°, from 25.4° to -24.2°, is too great. In the 11th Dynasty (-2055/-1985) and at the end of the 18th (-1550/-1295) the obliquity of the ecliptic would have been c. 23.9° and 23.8° respectively. Of course if the precise orientation of the temples was determined, not by the actual bearing, but by the associated triangle, then some discrepancy from the true rising azimuth is to be expected. The walls of a rectangular structure point in four different directions and at Abydos and Karnak similarly oriented temples are recorded as pointing to 26.75°. 116.75° and 296.75°.[207] Whatever was of interest at the horizon it is unlikely that they had found an orientation that would fit all three perfectly.

[205]Very few of the recorded values included fractional values less than 0.5°, indicating that when built their precision was probably not better than 0.25°. One of the nine, including the fraction 0.75°, was 48.75°, corresponding to an absolute 41.25°.

[206]The sides of the 'fort' are 64.7m x 56.7m, with a ratio of 8/7, but, unlike later temples at Karnak, a diagonal was not aligned with the meridian. Nekhen News 31.

[207]This absolute angle (26.75°) appears at Abydos c.-1550 as 26.75°, corresponding to a declination of 52.75° and at Karnak as 116.75° c.-1480, where it approximates to the rising of the mid-winter sun.

Previously we have noticed an interest in bearings with 'nice' cotangents, such as 2 (26.565°) and 3 (18.435°) and Table 36 shows that the eleven earliest temples had bearings that can be related either to simple cotangent ratios or to angles contained in Pythagorean triangles. Before the 11th Dynasty most of the bearings appear closer to having simple cotangent ratios.

Table 36 Eleven earliest temples, based on simple Cotangents or Pythagorean Triangles

Belmonte et al[208]	Location	Dynasty	Recorded Azimuth (adjusted)	Cotangent/(angle)	Diff.	Pythagorean triangle (angle)	Diff.
			degrees	ratio (degrees)	diff. degrees	side ratios (degrees)	diff. degrees
1/46	Thoth Hill	archaic	119.5 (29.5)	**9/5 (29.05)**	+0.45		
4/42	Kom el Ahmar	pre-dyn	44.5	**1/1 (45)**	-0.5		
4/41	Kom el Ahmar	pre-dyn	48.5 (41.5)	**8/7 (41.19)**	+0.31	48/55/73 (41.11)	+0.39
4/43	Kom el Ahmar	2	54.5 (35.5)	**7/5 (35.54)**	-0.04		
1/1	Abydos	2	46 (44)	**30/29 (44.03)**	-0.03	20/21/29 (43.6)	+0.4
4/24	Abydos	4	48.75 (41.25)	**8/7 (41.19)**	+0.06	48/55/73(41.11)	+0.14
1/48	Thoth Hill	11	117 (27)	2 (26.57)	+0.43	**105/208/233 (26.78)**	+0.22
1/49	Deir Bahari	11	118.25 (28.25)	11/6 (28.61)	-0.36	**8/15/17 (28.07)**	+0.18
4/38	W.Thebes	11	121 (31)	**5/3 (30.96)**	+0.04		
4/25	Abydos	12	26 (26)	2 (26.57)	-0.56	**39/80/89 (25.98)**	+0.02
1/83	Tod	12	145.5 (34.5)	3/2 (33.69)	+0.81	**160/231/281 (34.71)**	+0.29

Working with 'nice' cotangents has the major inconvenience of not having an exact integer length for the hypotenuse. Although they were capable of working out square roots, it was probably burdensome for an actual builder. If you only wanted a rough bearing, it would not be a major problem, but if you also wanted to lay out a rectangular building, you need to be sure your triangle contained an exact right-angle.

Temple Layouts at Karnak and the Mesopotamian Tablet Plimpton 322.

We have noted above that meridian lines ran diagonally across both the Festival Hall of Thutmose III and the Hypostyle Hall at Karnak. The triangulations implicit in such an arrangement are shown in Figure 21. It contains three different sizes of similar right-angled triangles. For reasons which will become obvious the sides of such triangles are described as b for the short side, l for the long side and d for the hypotenuse: the same notation as used by Neugebauer in respect of the Plimpton tablet.[209] Here the two middle-sized triangles are assumed to have sides with lengths b, l and d, with those for the smaller pair being reduced in the ratio b/l and the largest increased by d/l. This means that the ratio of the areas of the three different sized triangles are b^2/l^2, 1 and d^2/l^2 This latter ratio appears in the first extant column of the Plimpton tablet. Crucially it is also the ratio of the length (d^2/l) of the diagonal of the largest triangle to the long sides of the mid-sized triangles (l).

How could such a layout be set out? Firstly choose the desired row with its corresponding small angle. In figure 21, for simplicity, the ratio of the sides and diagonal of the desired rectangular building are 3/4/5, as in the Plimpton tablet row 11, with the sides inclined

208 The data is from Shaltout et al and Belmonte et al (see footnotes 180 & 192)

209 Neugebauer O., The Exact Sciences in Antiquity, Dover, 1969, p 36.

36.87° to a desired direction (for the two temples at Karnak it was the meridian). Secondly peg out a line (PS) in that direction and with the desired length, say 125 cubits, for the diagonal of the finished temple. This length would correspond to d^2/l in the units of row eleven, where the first extant column has d^2/l^2 as 25/16, which divided into 125 gives 80 cubits for l. This fixes points Q and R. Knowing the ratio of the three sides, b would be 60 cubits and d 100 cubits. Thirdly with all three sides of a triangle of the correct size, it would be a simple matter to determine the other two corners of the building. This strongly suggests that Plimpton 322 would have been useful to a surveyor wanting to set out buildings in this manner. That tablet is dated to the Old Babylonian period, which roughly corresponds to the 11/12th dynasties in Egypt and from Table 36 we can see a growing interest in Pythagorean triangles from Dynasty 11 onwards. Satisfaction with this conclusion must be tempered by our not knowing, for certain, what was in the missing part of Plimpton 322, but ideally it would have included the long side, corresponding to the short side and hypotenuse of the second and third extant columns, so that the surveyor would know the ratio of the units in the Plimpton row to his chosen cubit length of the diagonal. He could then apply that ratio to the units in the second and third extant columns to get their lengths in cubits and so determine the other two corners.

Mesopotamia - Minas, equilateral triangles and the 'pole' on the ground, including drawing a circle of desired circumference without knowing either the radius or Pi.

The term 'minas' commonly applies to weights and in his analysis of balance pan weights at Nippur Hafford recorded seven of one 'mina' with a mass between 481 and 500 and a mean of 496 gms.[210]

However in Mul-Apin (c.-1000) the ideal year had 360 days with a 'mina' corresponding to one sixth of a day, or 60° (time), and the longest and shortest days being 4 and 2 minas (ratio 2:1)[211] The same mina sub-divisions of the day can be seen in BM 17175 + 17284 from the Old Babylonian period (c.-1900).

In reality, on a latitude of c.35°, the longest and shortest days are, measured in time, 216° and 144° (ratio 3:2, recognised in Mesopotamia by -c.700) but if measured in azimuth they are 240° and 120° (ratio 2:1).[212] From time immemorial they would have seen that the rising and setting of the sun at the two solstices, combined with the meridian, divided the horizon into six equal equilateral triangles, which together would form a hexagon. It is not therefore too surprising that O'Connor and Robertson refer to a theory that 'an equilateral triangle was considered the fundamental building block by the Sumerians'.[213]

Neugebauer (1975) referred to 'the assumption that the given weights represent the outflow of water from the bottom of a cylindrical container the ratio 4:2 ratio is not in flagrant contradiction to the Babylonian standard M:m = 3:2…', although mul-Apin does

[210]Hafford, W. B., Mesopotamian Mensuration Balance Pan Weights from Nippur, JESHO 48,3, Brill, 2005.
[211]Hunger H. and Pingree D., MUL.APIN An Astronomical Compendium in Cunieform, Archiv fur Orientforschung, Beiheft 24, 1989.
[212]For both these two ratios to be precisely correct the latitude of the observer must have been 34.25°, calculated from $\sin \phi = \tan 60 / \tan 72$, but in practice a latitude within 2° or 3° of this value may well have been acceptable. For a latitude of 30° the sine is ½, which could hardly be more simple and, in Egypt, Heliopolis and the Great pyramid are on latitudes of 30.11° and 29.98° so it is not surprising that in Egypt they seem to have adopted horizontal dialling at an early date. In northern Mesopotamia the sine of 34.85° is 4/7, not quite such a simple ratio and unattractive in a sexagesimal system.
[213]O'Connor J.J. & E.F. Robertson, Babylonian Numerals, University of St. Andrews, 2000

not mention water or clocks.[214] It seems inherently unlikely they would not have considered the two ratios contradictory after they had determined the more accurate 3:2 ratio. However before then they might well have thought that a water-clock gave a ratio which was close enough to the long-established 2:1 ratio in azimuth, This would have provided a justification for associating time with a weight of water even after they were using dialling and not water-clocks. To-day we happily use horse-power long after horses have ceased to be a main source of power.

In AO6478 (c. -700) Hunger & Pingree record 60 2/3 minas for a year of 364 days, but state categorically that 'The weights (of water) regulate a water-clock in which 1 mina of water measures 0;1 days in contrast to the older tradition which used a water-clock in which 1 mina measured 0;10 days', a ten-fold difference.[215] Neugebauer's assumption had become an accepted fact. Modern studies of water-clocks were discussed above (p.45) and it would be helpful if we could find another explanation for the use of 'minas' which is less dependent on water-clocks.

We have noted the traditional ratio of 2:1 for the longest and shortest days of the year, measured in azimuth, but at a distant horizon azimuth angles cannot be distinguished from the hour-line angles around the 'pole' on the ground. At Hierakonpolis in Egypt there was a possible early interest in this 'pole' around -3000, which was later clearly manifest in the layout of the pyramids, particularly that of Userkaf (c.-2500) (see p.17ff). In the beginning it would have been noted that the 'shadows' of rows of stars rotated about the pole on the ground, but only later would they have managed to distinguish different equinoctial times between such rows. In Mesopotamia there is little early evidence of an interest in the 'pole' on the ground, but the GU text (c.-700) had probably 30 strings of stars with similar R.A. whose 'shadows' would rotate about that 'pole'.[216] Even if we assume that the Astrolabe Texts were an earlier manifestation of an interest in the 'pole', that only takes us back to the end of the second millennium B.C. With little, except possibly at Assur (see below), apparent interest in the 'pole' on the ground, the reign of the 'water-clock' may have lasted longer in Mesopotamia than in Egypt, thus leading to a close association of weights of water with the 2:1 ratio in azimuth, with each mina corresponding closely enough to 60° of azimuth. When dialling around the 'pole' was adopted the same term was retained for 60° of hour- angle.

We have already shown (p.3) that the observers of the proximity of planets to the Normal stars moved along a 'stepped' curve where each north/south cubit corresponded to 2.5° (azimuth). At that time a mention of cubits in a celestial context might have automatically implied the use of that curve. Similarly a mention of 'minas' might have implied the use of 60° angles around the 'pole' on the ground. In other words the terms 'cubit' and 'mina' may have defined the type of instrument being used. The tenfold change in the mina, noted by Hunger and Pingree can be readily visualised if we assume that it was due to an increase in the size of the measuring instrument, rather than a change in the unit of measurement.

Figure 22 shows a small hexagon, centred on a gnomon, with sides of one unit, appropriate for a day of six minas, as in Mul-Apin. And also two others, ten and sixty times larger, centred on the 'pole'. Equilateral triangles have an interesting property: After dividing a side of the largest hexagon into ten equal parts, a circle through the ends of the

[214]Neugebauer, O. A History of Ancient Mathematical Astronomy, Springer, 1975, p 708..
[215]Hunger H. and Pingree D., Astral Sciences in Mesopotamia, Brill, Leiden, 1999, p.85
[216]Hunger H. and Pingree D.,(1999) op. cit., p.90.

first sub-divisions nearest an apex has a circumference of 359.6 units. Without knowing either Pi or the radius, it is possible to construct a circle of any desired circumference with a precision of just over 1 per mil.[217]

From Mul-Apin, at the beginning of the first millennium BC, we know of their interest in zigpu stars, which transit at the zenith and ideally have a declination equalling the geographic latitude of the observer.[218] On a latitude c. 34.75° such stars rise 118.8°(time) before transit on an azimuth of 46.1°. For these values to be precisely 120° and 45°, the observer must have been closer to a latitude of 35.26°, which is 1° greater than that calculated for the 2:1 and 3:2 ratios for the longest/shortest days. The average of the two calculated latitudes is 34.75°, which is comparable to those of Mari, 34.54° and Assur 35.45°. At that average latitude the 'shadow' of a zigpu star crosses the main 30° hour-line divisions of 120°, 90°, and 60°, at altitudes of 6.5°, 19.0° and 32.4° with respectively c.108°, 90°, and c.72° of time to transit . The cardinal positions of the sun at the horizon would lead to a subdivision of 18° in time or subdivisions thereof, but their interest in zigpu stars either led to or coincided with the adoption of the us, a time-degree, with 30 such degrees in a beru. Seemingly in northern Mesopotamia, observations of the sun's movements, on their own, did not lead to the adoption of nice equinoctial sub-divisions of time, which the Egyptians were using by the time of the pyramids, a millennium and a half earlier (see p. 28ff).

At Assur two sides of the Temple of Anu and Adad are oriented c. 44°/224°from magnetic north, so would align with a zigpu star, on the horizon, and the Temple of Ishtar has two sides c. 59°/239°, close to the rising and setting of the sun in mid-summer and mid-winter respectively.[219] Like other plans of these temples only magnetic north is shown, but at the time of the original archaeological work there in 1910/13 the magnetic declination was about 2° east, so the difference between magnetic and true north was small.[220] With this caveat it seems that the building of these two temples confirms their interest in horizon phenomena, and it would be a relatively short step to use the rotation of the 'shadows' of celestial objects around the 'pole', to estimate the passage of time, even if they did not have the advantage given to Egyptians on the 30th parallel, where sine 30° equals ½.[221]

Rogem Hiri

There is a dearth of material, suitable for analysis, at this site making it difficult to date. What follows includes an attempt to date its radial lines, using techniques recorded elsewhere in this appendix, such as the preference for angles with nice tangents.

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[217]Construct an equilateral triangle with sides one-sixth of the desired circumference and subdivide one side into 10 equal lengths. Draw a circle through the inner ends of the two outside subdivisions, nearest an apex, and centred on the opposite apex. For a circumference of 360° its radius would be 57.236 units, which divided into 180 gives 3.145 for Pi. This observation is consistent with Robson's conclusion that ' circle was the shape contained within an equidistant circumference' Robson E., Words and Pictures: New Light on Plimpton 322, The American Mathematically Monthly, 2002, Vol. 109, pp105/120.

[218]Hunger and Pingree (1989) op.cit, p. 143. Table V lists 13 zigpu stars, which were identified as stars with a declination ranging from 31.71° to 44.28° and a mean of 36°, which gives an indication of their tolerance for less than precise values.

[219]The alignments were measured on Figure 46 in Lloyd S. and Muller H.W., Ancient Architecture, Faber & Faber, 1986, p.30.

[220]The magnetic declination was calculated using NASA's Magntic Field Calculator Model IGRF

[221]On latitude 30° the ratio of the tangents of the time to transit and the hour-line angle to the meridian is the simple 2:1.

The monument lies between the River Jordan and the Golan Heights almost due east of the northern end of the Sea of Galilee.[222] The area immediately around the site has a fall, from east to west, of 10m over about 520m.[223] A cross-section (Figure 24) shows the site itself as close to level from the centre eastwards, but with a significant drop to the west. An observer standing at the base of the tumulus would see the western outer dry stone wall as well below the horizon, whereas in the east it was perhaps only marginally below, say less than 1°.

The present tumulus resulted from the collapse of a stepped cone, with an original height of over 4m, which was 'built on a naturally elevated basalt formation'.[224] Under the tumulus there is a later burial chamber and under that an unworked basalt slab 'oriented north-east/south-west, as the direction of the dromos' so from the lowest levels we have evidence of the importance of this axis.[225] We can imagine the first 'surveyor' standing on the slab and instructing his workers to place rocks on strategic alignments. At that stage the site, although much smaller, would have resembled Nabta Playa in Upper Egypt.[226] The tomb axis is given as 58.12°, very close to 57.99° a 'nice' angle because its tangent is 8/5. In the opposite direction it would be 238.12°, corresponding to the setting of Sirius c. -4000.[227]

There are three outer concentric walls, labelled 2, 3 and 4 in figure 24, although others have numbered them 3, 2 and 1.[228] Number 2 is relatively modest with a thickness between 1.8 and 2.0m and a preserved height of about 1m. Its perimeter has a bulge on the south-west and a kink on the south-east. Walls 2 and 3 appear to have been laid out around the same point some 3.2m south of the centre of the tumulus.[229] Wall 4 is the most impressive with a thickness of 3.2/3.3m and a current height of c.2.5m. Its centre was about 4.25m south of the burial chamber and later we will suggest a justification for this shift.[230]

One of the difficulties with the site is a dearth of dateable material. Friekman and Porat suggest the Chalcolithic period for its initial development, largely based on the local population density but they also found sediment samples, dating to the 4th millennium BC. [231] In the second half of that millennium the second brightest star, Canopus (α Car), became visible due south. There were also two major conjunctions in declination on -24° between Sirius (α CMa), the brightest star, and the sun at the winter solstice (c.-3400) and on +24° between Capella (α Aur) and the sun at the summer solstice a little later (c.-3350).

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222 Google Earth: Lat. 32.81°, Long. 35.80 Est, Altitude 516m

223 Zohar M., A Megalithic Monument in the Golan, Hebrew University of Jerusalem, JSTOR 27926134, 1989, Fig.3 (reproduced here as figure 24)

224 Mizrachi Y., Mystery Circles, Biblical Archaeological review, Vol. 18, no.4, 1992, p.50; his figure on p.49 reproduced here as figure 25. See also Mishrachi et al, The 1988-1991 Excavations at Rogem Hiri, Golan Heights, Israel Exploration Journal, 46 (3-4) 1996, pp 177/8 .

225 Mizrachi et al, op.cit., p.180 refers to an 'earlier construction phase…. At the centre of the complex'.

226 Malville J.McK, Schild R., Wendorf F. and Brenmer (sic) R.,Astronomy of Mabta Playa, African Skies 11, July 2007.

227 SkyMap Lite 2005.

228 Zohar, op.cit. p.23, recorded the main dimensions of the three outer walls and a preliminary inspection suggests they were more egg-shaped than circular, with the pointed ends to the south

229 Aveni A. and Mizrachi, Y, The Geometry and Astronomy of Rujm el-Hiri, a Megalithic site in the southern Lavant, Journal of Field Archaeology, 25(4),1998.

230 The 4.25m is derived from the centre assumed for the calculations in Table 1 of Aveni and Mizrachi. For the two equinox marker stones the angular difference for the two centres is 3.5°, which at distance of 69.5m equates to 4.25m.

231 Freikman M. and Porat N., Rujm el-Hiri: The Monument in the Landscape, Hebrew University of Jerusalem, Geological Survey of Israel, Tel Aviv, 2017, Vol. 44, pp14-39. For sediment analysis see their table 3. On page 27 they admit the possibility that construction may predate -4000.

Aveni and Mizrachi noted that the setting of Sirius was directly opposite the north-eastern entrance way.[232] This indicates that the monument could be used from the centre looking outwards and also in the opposite direction. In a theoretical flat landscape celestial objects with a declination of ±24° would rise on azimuths of 90°± 29°.

The sun casts shadows of anything in its path but to capture the 'shadow' of a star the observer would need to keep it aligned with the top of a gnomon. This is awkward because he ought to have his eye at ground level like the shadows cast by the sun. We have shown above that at Hierakonpolis, c-3400, they seem to have used low fences to follow 'shadows' of stars and were beginning to show an interest in the rotation of stars around the 'pole' on the ground.[233]

The details of the two entrances in the north-east and south-east are summarised in Table 37. Ignoring any departure from a flat horizon, the azimuths of the sides of the northeastern entry equate to declinations of about 16° and 28°, a range of 12°, suggesting it was designed to accommodate the rising of the moon and planets, as well as the sun, around the summer solstice.

The basic north-east/south-west orientation had a simple tangent of 8/5, so perhaps other recorded angles had similarly nice tangents, as we have noted elsewhere in the region. In Table 37, column C, the azimuths for the outer wall opening to the north-east have tangent ratios of 3:1, 2:1 and 3:2 three of the simplest ratios, which was certainly no coincidence. Furthermore the corresponding azimuths for the south-eastern opening are nearly as nice. Evidently the layout of both openings was based on azimuths around point 79, above the burial chamber, and not around the geometric centre of the walls.

[232]Aveni and Mizrachi op.cit. p.489.
[233] Bremner R.W., Babylon Linear Measures of Celestial Angles and an Observatory, 2023, web site British Astronomical Association pp 17/27.

Table 37 Data from Aveni and Mizrachi, Table 1. N.B The outer values for wall 1 in the north-east appear to have been interchanged and in the south-east the values for the centre and southern side may have been similarly switched. Walls referred to as 1 and 2 by Aveni and Mizrachi are labelled 3 and 4 by Zohar in figure 24.

	Azimuth measurements around point 79, above tomb					Calculated values around geometric centre						
	Azimuth	Tangent	Tangent fraction	Corresp-onding azimuth	Diff. A less D	Bearing angle	Tangent	Tangent fraction	Corresp-onding bearing	Diff. F less I	Calc. rising to transit	Tangent time
NE entry	A	B	C	D	E	F	G	H	I	J	K	
	degrees	ratio	ratio	degrees	degrees	degrees	ratio	ratio	degrees	degrees	degrees	
Wall 2 N	55.95	1.48	3/2	56.31	-0.36	53.83	1.37	7/5	54.46	-0.63	68.37	2.52
Wall 2 C	58.00	1.60	8/5	57.99	0.01	55.88	1.48	3/2	56.31	-0.43	69.85	2.73
Wall 2 S	59.97	1.73	7/4	60.26	-0.29	57.95	1.60	8/5	57.99	-0.04	71.25	2.94
Wall 1 N	70.83	2.88	**3/1**	71.57	-0.74	68.43	2.53	5/2	68.2	0.23	77.88	4.66
Wall1 C	63.23	1.98	**2/1**	63.43	-0.20	60.83	1.79	9/5	60.95	-0.12	73.12	3.30
Wall 1 S	55.65	1.46	**3/2**	56.31	-0.66	53.25	1.34	4/3	53.12	0.13	67.93	2.47
SE Entry												
Wall 1 N	140.73	-0.82	5/6	140.19	0.54	139.28	-0.86	5/6	140.2	-0.92	122.28	1.58
Wall 1 C	161.10	-0.34	1/3	161.58	-0.48	160.18	-0.36	1/3	161.57	-1.39	146.47	0.66
Wall 1 S	152.78	-0.51	1/2	153.43	-0.65	151.85	-0.54	1/2	153.43	-1.58	135.17	0.99
Equinox stones												
	86.78					83.28						
	94.35					90.85						

In -3000 a Cmi (R.A.49°) and a Sco (R.A. 179°) rose on azimuths of 86.13° and 94.37°, closely matching the positions of the 'equinox' stones, which suggests that by -3000 this bearing had been recognised, but we cannot take this as dating the building of the wall for the large marker stones could have been placed on the ground at that time and later raised to sit on top of the wall.

It is not clear what north is indicated on plans of the site. For instance Aveni and Mizrachi's figure 2 merely says north is to the top, but does not state if it is magnetic or true north. In Figure 25 we have made an anti-clockwise adjustment of 5.91° to the indicated north. This value is not too far from the present magnetic declination in Israel, but is double what it must have been around 1990, unless there is some local anomaly around Rogem Hiri.[234] This adjustment in figure 25 brings the azimuths on the plan into line with the azimuth measurement for the southern side of the north-east opening in Table 37 and it also seems consistent with the equinox marker stones being due east of the 'pole'[235]. The layout of the two entry points can be visualised as being on a 12 x 12 grid, with each grid square having sides of 11.7m (figure 25)

[234]For historic magnetic declination see Survey of Israel, Shirman B., 40 years of magnetic observatories and 100 years of magnetic declination measurement in Israel, 2016.
[235]Measurements on a Google Earth Map suggests 5.91° may be too high and 3.31° would be closer.

Central Structure.

If, as suggested, the outer wall in the east was intended to be just (say 0.5°) below the horizon we can estimate that the gnomon at the centre must have been c.0.65m above the level of the wall, with its height above ground level depending on the height of that wall. Aveni & Mizrachi indicate that the present height of the outer wall is 'more than over 2m' in places, but in the later paper it is given as up to 2.5m.[236]Also unless the large equinox marker stones were only the base of an even higher structure the intended height cannot be much above what now remains, suggesting a height at the centre in the region of 2.65 and 3.15m.

Assuming there was a small gnomon at the centre, Table 38 sums up significant points, where the sun's shadow would cross the meridian, together with the position of the celestial pole on the ground.

Table 38 Sun's shadows on the meridian for Latitude 32.9° with the obliquity of the ecliptic 24°

	Altitude	Positions on meridian		
Gnomon Height metres		1	2.75 (b)	4.0 (c)
	degrees	metres	metres	metres
Winter solstice	33.1	1.534 N (a)	4.2	6.1
Equinox	57.1	0.647 N	1.8	2.6
Summer solstice	81.1	0.157 N	0.4	0.6
Gnomon		0	0	0
Pole on ground	32.9	1.546 S (a)	4.25	6.2

Notes a. Around -5700 Obliquity of ecliptic 24.2° and both mid-winter shadow and the 'pole' would be 1.55 units north and south of a gnomon with unit height.
b. 2.75m is calculated from the distance of the geometric centre from the gnomon.
c. Height of stepped cone

Aveni and Mizrachi showed the centre of the three outer circular walls some metres south of the burial chamber, with the centres of inner walls further north under the tumulus.[237] There is a possible justification for this shift. A central cairn, 2.75m in height, would be just a little above the height of the outer wall and would have the 'pole' 4.25m to the south and the winter sun would cross the meridian 4.20m north. Around -3400 at Hierakonpolis in Egypt they were showing an interest in lines rotating about the 'pole'. Here a gnomon of 2.75m must have pre-dated the stepped cone of 4m. On the other hand the centres for walls 2 and 3 were c.3.2m from the burial chamber or some 25% closer than the centre of the outer wall. If these walls too were centred on a 'pole', the height of the gnomon must then have been closer to c.2.0m.

It seems reasonable to believe that the later building of the 4m stepped cone coincided with the building of the square structures within the entrances. As the latter were said to be higher than the nearby walls, we might assume the tops of the square structures were

[236] Aveni and Mizrachi op.cit. p.477
[237]Aveni and Mizrachi op.cit., p.479, figure 3.

much the same as that of the 4m cone, so that there would have been horizontal lines of sight between the three positions.[238]

Radial Walls

Eight walls between the second and third largest concentric walls were labelled A to H by Zohar.[239] Although generally described as radial, D, E & F are definitely not, but they do draw attention to three significant stars in the south. In -3500 Canopus, α Car, would have set and β Carina and β Centaurus would have risen within just seven minutes. By -3200 Canopus would spend two hours above the horizon. Watching the nightly movements in the far south would have stimulated an interest in time with one star, for example, crossing from rising to setting while another only reached the meridian. By -2600 α Car and β Car would take 1.5 and 2.0 hours respectively from the horizon to the meridian. Over this near millennium interest in these stars could have shifted from their simultaneous positions on the horizon to their time to transit, but we do not know when this shift occurred, so in figure 25 we have shown the calculated positions of specific hour-lines, defined by the simple tangent ratios in column f of table 39. The radial lines with nice tangents are close to the radial wall alignments, which mark the passage of time in units of 15° or simple subdivisions thereof, but there are anomalies. For wall F, which is probably associated with Canopus, the line is far from radial, with the inner end aligned with the meridian and the outer end with the one hour hour-line. From this we can conclude that those walls which are obviously not radial were probably intended to measure different times at either end, with the inner end being earlier, when the appearance of Canopus had only recently been recognised.

Table 39 Calculation of the hour-line angles, which are symmetrical north and south of due east, as in a horizontal sundial, with selected matching stars.

Hours to meridian	Decl. + or -	Azimuth	Hr-line to meridian	Tangent hour-line	Suggested fraction	Possible star & apparent brightness	Rising to transit time		Radial Line
							-3000	-2500	
a	b	c	d	e	f	g	h	i	j
	degrees	degrees	degrees	ratio	ratio		hours	hours	
1	-56.19	171.7	8.28	.145	1:6				
1.5	-55.00	167.32	12.68	.225	2:9	α Car -0.72	1.21	**1.55**	F
2	-53.24	162.6	17.41	0.314	1:3	β Car 1.68	2.26	**1.98**	
3	-47.54	151.5	28.51	0.543	1:2	α Tri 3.41	3.36	**3.02**	E
3.5	-43.26	144.7	35.29	0.708	7:10	α Lyr 0.03	8.64	**8.49**	H
4	-37.7	136.7	43.25	0.941	1:1	α Cyg 1.25 α Cen 5.15	**7.93** 4.25	**7.93** **4.05**	A D
5	-21.81	116.3	63.74	2.027	2:1	α CMa -1.46 β Gem 1.14	**5.00** **7.08**	5.08 7.18	G B
5.5	-11.41	103.6	76.38	4.127	4:1	γ Ori 1.64	**5.54**	5.66 (a)	C
6	0	90	90						

Note a. We have assumed γ Ori was associated with a time of 5.5 hours, but perhaps it should have been 5.67.hours

Further north walls A, B, G and H are close to the respective hour-lines, while C shows a modest shift in radial angle between the inner and outer ends, suggesting that the intended

[238]Zohar, op.cit p.24

[239]Zohar, op.cit,, his figure 3 shown here as figure 24.

times were not identical. Their knowledge of hour-lines did not come from a learned study of the theory of sundials but from observation of specific stars and Table 39 includes suggestions of which they might have been and the nine stars include four of the five brightest. For the radial walls as a group their date can be considered as one or two decades before -2500, when all the stars were within 5 minutes of the respective target time, with one exception β Gem (11minutes, wall B)[240] They were early adopters of a 15° hour.

It would seem the radial walls were designed to lie in the narrow gaps between lines of azimuth, around a gnomon, on the north side and hour-lines, around the 'pole', on the south.[241] An accurate survey would reveal if the alignments had 'nice' tangents or were actually determined by direct observation of objects on the horizon

[240]Wall C is associated with a time of 5.5 hrs, but making a small adjustment to 5.67 hrs would agree with a date of -2500.
[241]The two lines of azimuth and hour-line meet at infinity or a distant horizon.

Table 40.Proposed Time line – Rogem Hiri

Approx. date	Event	Gnomon Ht
Landscape features	Mt Hebron azimuth 359°, altitude 2.8° Tel Fares azimuth 47.3 °, altitude 2.7°, decl. 36.5° Mt. Tabor azimuth c.236 °, altitude ? decl. c.-28°	n/a
-5700	Obliquity of the ecliptic 24.2° Shadow of mid-winter sun and the 'pole' would be 1.55 units north and south respectively of a gnomon of unit height	
-4360	Sirius sets by Mt Tabor	
-4000	Sirius sets on 238.3°opposite 58.3° on main site axis	
-3500 (a)	Sirius, α CMa, sets opposite centre of NE entryway Canopus, α Car, just visible due south	
-3400	Sirius on same path as sun at Winter solstice	
-3350	α Aur on same path of sun at Summer solstice Sirius and a Aur rise & set opposite each other	
-3200	Canopus above horizon for two hours	
-3000	Sirius 5 hours from horizon to transit	
-2600	Canopus above horizon for three hours	
Pre -2500	Construction of 3rd largest circular wall	
Just before -2500	9 Selected stars, except β Gem, within 5 minutes of specified time, construction of radial walls between 2nd and 3rd largest circular walls and construction of 2nd largest circular wall	
Post -2500	Construction of largest circle with height just below top of gnomon	c.2.75m
later	Construction of square structures within entrances, construction of stepped cone at centre	4m
1400	Sirius now too high in the sky to set opposite the NE entrance in outer wall	

Summary Timeline

Egypt in red

Year	Location	Pythagorean Triangles etc	Subdivisions of circle	Time
-5500	Tell es-Sawwan		45°	
-5100	Eridu		30°	
-4700	Nabta Playa	3,4,5		
-4500	Egypt			Year 360 +5 days
-4450	Nabta Playa		26.56°	
-4250	Eridu	3,4,5		
-3900	Abydos		5 pointed star	72° divisions
-3400	Hierakonpolis	3,4,5 5,12,13 9,40,41		Discovery of 'Pole' on ground
-3100	Mesopotamia		8 pointed star	
-3000	Egypt		spirals	
-2600	Saqqara	4 later 6 different		Horizontal Dials
-2556	Khafre's pyramid Giza			Built-in hours 60 minutes
-2500	Rogem Hiri		Nice tangents	Hour-lines
-2500	Menkaure's Causeway?			Three season year
-2500	Standard Pyramids	3,4,5		Standard hour 60 minutes & short hour 40 minutes
-2300	Coffin Lids Many ex Asyut			Short hour 40 minutes
-1900	Mesopotamia Old Babylonian Period	26 different - Ark & Plimpton tablets	Circle drawn without knowing radius or Pi	daylight 2:1 & 3:2 ratios & interest in zigpu stars & 30° division (beru & us) of time
-1800	Thebes		Horus Eye fractions	
-1500	Egypt			L-shaped sundials
-1300	Upper Egypt Abydos	Rt. triangles side 60		Djed Pillar E/W Sundial
-1000	Mesopotamia Mul-Apin			Shadow length table
-700	Babylon		azimuth in 2.5° steps	Longitude near horizon
-400?	Babylon			Ready Reckoner for converting rising azimuth to rising time

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